数字信号处理

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第五章 数字滤波器

FIR数字滤波器

频率取样设计法

设计原理

$$H(k)=H_d(k)=H_d(z)\Big|_{z=e^{j(\frac{2\pi}{N})k}}$$

$$=H_d(e^{j\omega})\Big|_{\omega=(\frac{2\pi}{N})k}$$

内插公式

$$H(z)=\frac{1-z^{-N}}{N}\sum_{k=0}^{N-1}\frac{H(k)}{1-W_N^{-k}z^{-1}}$$

$$\begin{cases} H(z)=\sum_{k=0}^{N-1}H(k)\Phi_k(z) \\ \Phi_k(z)=\frac{1}{N}\frac{1-z^{-N}}{1-W_N^{-k}z^{-1}} \end{cases}$$

设计原理

$$z = e^{j\omega}$$

$$\begin{cases} H\left(e^{j\omega}\right) = \sum_{k=0}^{N-1} H(k)\,\Phi_k\left(e^{j\omega}\right) \\ \Phi_k\left(e^{j\omega}\right) = \dfrac{1}{N}\dfrac{1-e^{-jN\omega}}{1-e^{-j(\omega-k\frac{2\pi}{N})}} \end{cases}$$

$$\Phi_k\left(e^{j\omega}\right) = \frac{1}{N}\frac{\sin\left(\frac{N\omega}{2}\right)}{\sin\left(\frac{\omega}{2}-\frac{\pi k}{N}\right)}e^{-j(\frac{N\omega}{2}-\frac{\omega}{2}+\frac{\pi k}{N})}$$

$$\Phi_k\left(e^{j\omega}\right) = \Phi\left(\omega - k\frac{2\pi}{N}\right)$$

$$\Rightarrow \Phi(\omega) = \frac{1}{N}\frac{\sin\left(\frac{N\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}e^{-j\left(\frac{N-1}{2}\right)\omega}$$

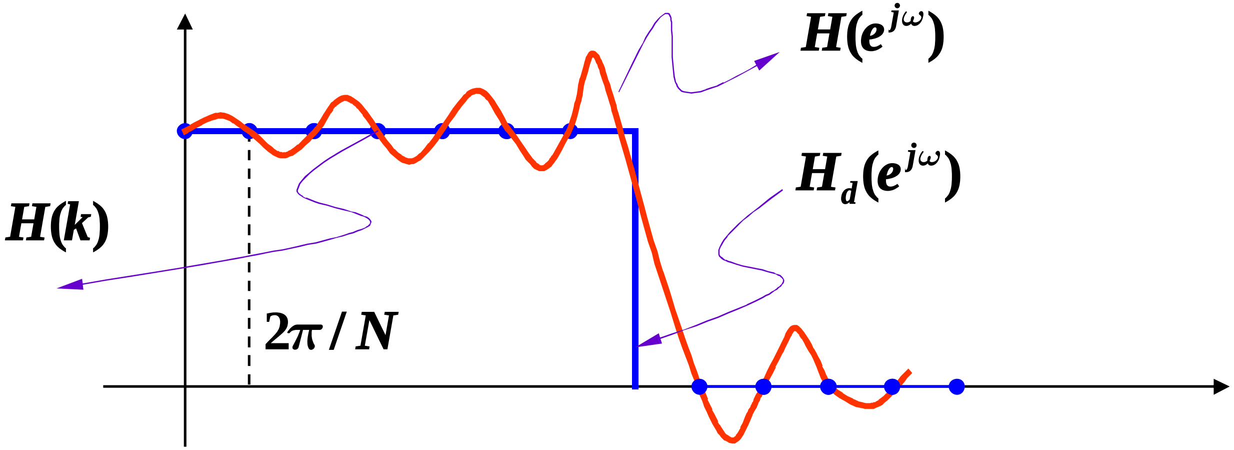
$$\Rightarrow H\left(e^{j\omega}\right) = \sum_{k=0}^{N-1} H(k)\,\Phi\left(\omega - k\frac{2\pi}{N}\right)$$

$$= \frac{e^{-j\left(\frac{N-1}{2}\right)\omega}}{N}\sum_{k=0}^{N-1} H(k)\left[\frac{\sin\left(\frac{N\omega}{2}\right)}{\sin\left(\frac{\omega}{2}-\frac{\pi k}{N}\right)}e^{-j\frac{\pi k}{N}}\right]$$

$$H\left(e^{j\omega}\right)=\sum_{k=0}^{N-1}H\left(k\right)\Phi\left(\omega-k\frac{2\pi}{N}\right)=\frac{e^{-j\left(\frac{N-1}{2}\right)\omega}}{N}\sum_{k=0}^{N-1}H\left(k\right)\left[\frac{\sin\left(\frac{N\omega}{2}\right)}{\sin\left(\frac{\omega}{2}-\frac{\pi k}{N}\right)}e^{-j\frac{\pi k}{N}}\right]$$

$$s\left(\omega,k\right)=\frac{1}{N}e^{-j\frac{\pi k}{N}}\frac{\sin\left(\frac{N\omega}{2}\right)}{\sin\left(\frac{\omega}{2}-\frac{\pi k}{N}\right)}\Rightarrow\boxed{H\left(e^{j\omega}\right)=e^{-j\left(\frac{N-1}{2}\right)\omega}\sum_{k=0}^{N-1}H\left(k\right)s\left(\omega,k\right)}$$

内插函数



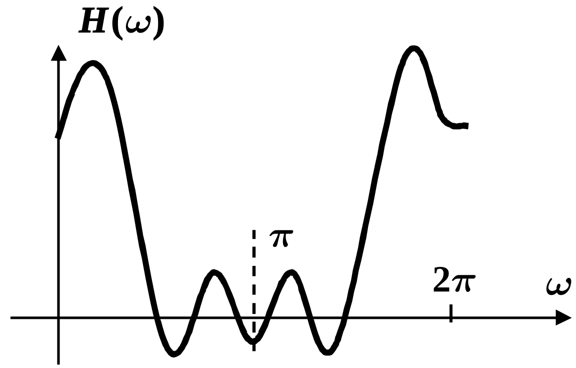
- 抽样点上，频率响应严格相等
- 抽样点之间，加权内插函数的延伸叠加
- 变化越平缓，内插越接近理想值，逼近误差较小

线性相位约束条件

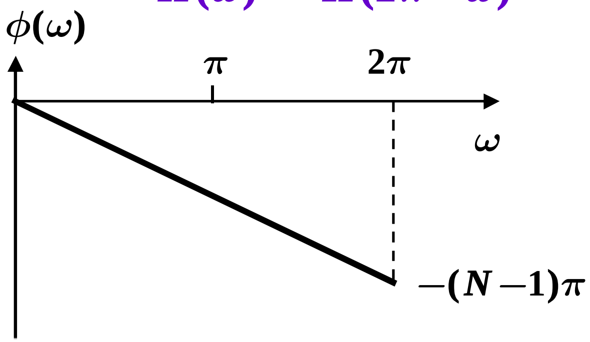
对第一类线性相位滤波器，h(n)为偶对称，N为奇数

$$H(e^{j\omega}) = H(\omega)e^{j\theta(\omega)}$$

$$\begin{cases} H(\omega) = \displaystyle\sum_{n=0}^{\frac{N-1}{2}} a(n)\cos[\omega n] \\ \theta(\omega) = -\left(\dfrac{N-1}{2}\right)\omega \end{cases}$$

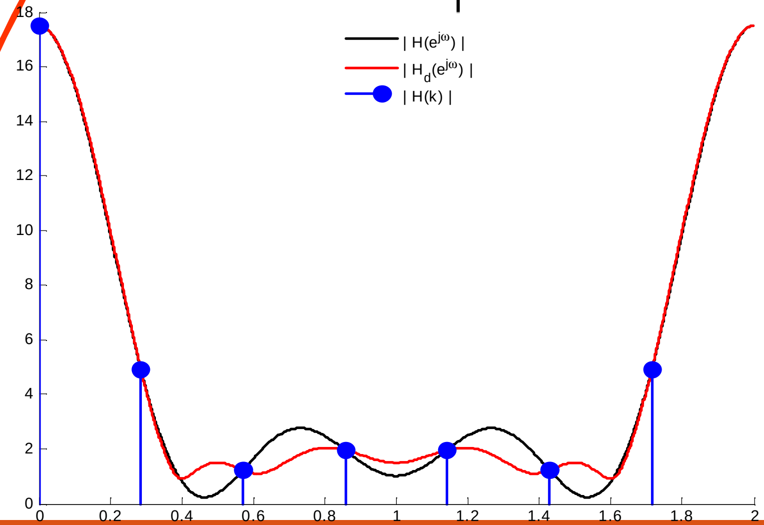


$$H(\omega) = H(2\pi-\omega)$$



$$H(k) = H(e^{j\omega})\Big|_{\omega=\frac{2\pi}{N}k} \quad k = 0,1,2,\cdots,N-1$$

$$\begin{cases} H(k) = \boxed{H_k} e^{j\theta_k} \\ \theta_k = \theta(\omega)\Big|_{\omega=\frac{2\pi}{N}k} = -\left(\dfrac{N-1}{2}\right)\dfrac{2\pi}{N}k \end{cases}$$



$$H(\omega) = H(2\pi-\omega)$$

$$\Rightarrow \boxed{H_k = H_{N-k}}$$

N为偶数:

$$H_k = -H_{N-k}$$

A summary:

线性相位 FIR 数字滤波器特点

① 时域：

$$h(n) = \pm h(N-n-1)$$

② 频域：

$$H(e^{j\omega}) = H(\omega) e^{j\left(\frac{L}{2}\pi - \frac{N-1}{2}\omega\right)}$$

$$H(z) = (-1)^L z^{-(N-1)} H(z^{-1})$$

— $H(\omega)$为实函数

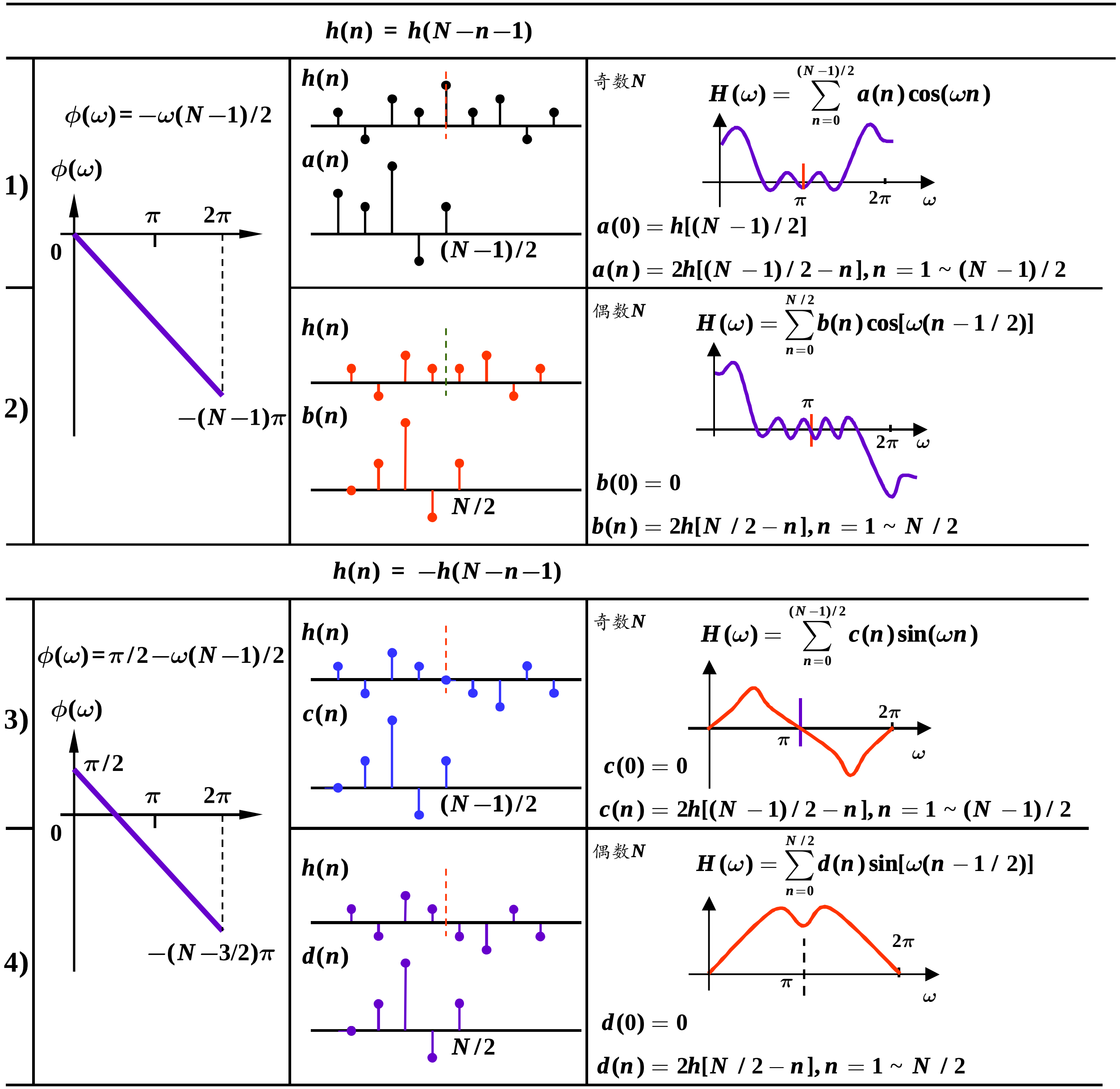
— $h(n)$偶对称：$L = 0$

— $h(n)$奇对称：$L = 1$

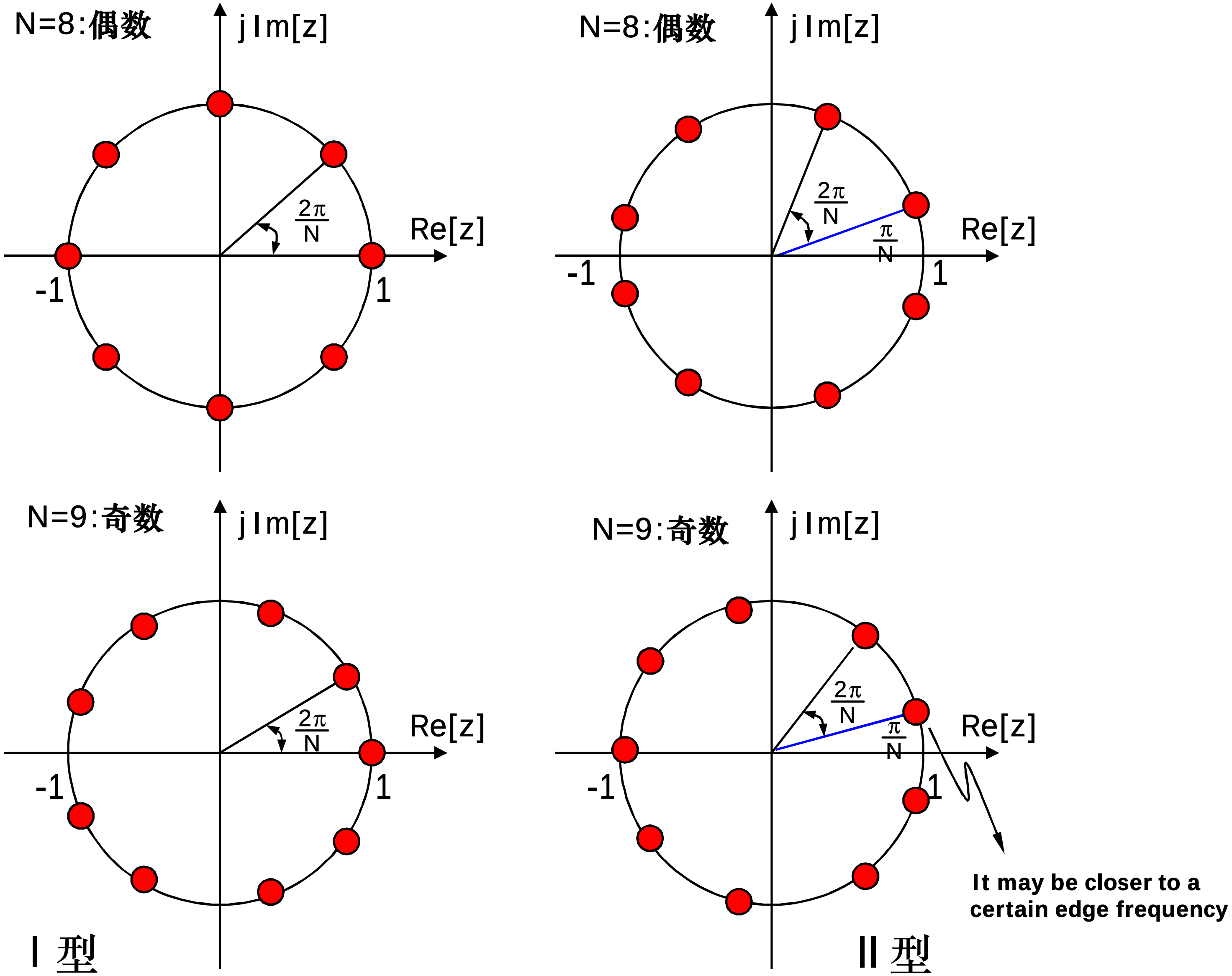
③ 零点：

成倒易对出现

线性相位FIR滤波器的四种情况



频率抽样两种方法

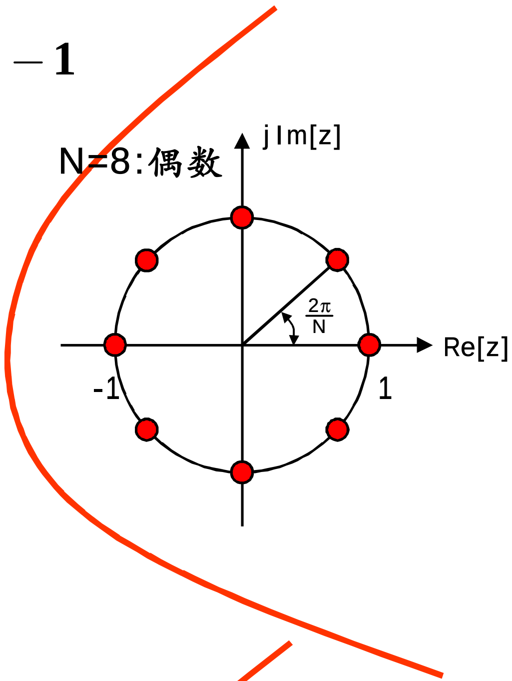

N=8:偶数
jIm[z]
Re[z]
$\frac{2\pi}{N}$
-1
1
N=8:偶数
jIm[z]
Re[z]
$\frac{2\pi}{N}$
$\frac{\pi}{N}$
-1
1
N=9:奇数
jIm[z]
Re[z]
$\frac{2\pi}{N}$
-1
1
N=9:奇数
jIm[z]
Re[z]
$\frac{2\pi}{N}$
$\frac{\pi}{N}$
-1
1
It may be closer to a
certain edge frequency
I 型
II 型

1）第一种频率抽样

$$H(k)=H_d(k)=H_d(z)\Big|_{z=e^{j\frac{2\pi}{N}k}}=H_d(e^{j\omega})\Big|_{\omega=\frac{2\pi}{N}k}\qquad k=0,1,\dots,N-1$$

系统函数： $$H(z)=\frac{1-z^{-N}}{N}\sum_{k=0}^{N-1}\frac{H(k)}{1-W_N^{-k}z^{-1}}$$

频率响应： $$H(e^{j\omega})=\frac{1}{N}e^{-j\left(\frac{N-1}{2}\right)\omega}\sum_{k=0}^{N-1}H(k)e^{-j\frac{\pi k}{N}}\frac{\sin\left(\frac{\omega N}{2}\right)}{\sin\left(\frac{\omega}{2}-\frac{\pi k}{N}\right)}$$



2）第二种频率抽样

$$H(k)=H_d(z)\Big|_{z=e^{j(\frac{2\pi}{N}k+\frac{\pi}{N})}}=H_d(e^{j\omega})\Big|_{\omega=\frac{2\pi}{N}k+\frac{\pi}{N}}\qquad k=0,1,\dots,N-1$$

系统函数： $$H(z)=\frac{1+z^{-N}}{N}\sum_{k=0}^{N-1}\frac{H(k)}{1-e^{j\frac{2\pi}{N}\left(k+\frac{1}{2}\right)}z^{-1}}$$

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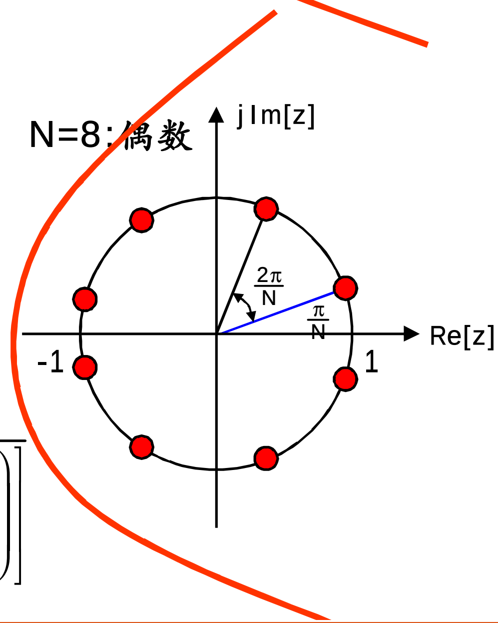
频率响应： $$H(e^{j\omega})=\frac{\cos\left(\frac{\omega N}{2}\right)}{N}e^{-j\left(\frac{N-1}{2}\right)\omega}\sum_{k=0}^{N-1}\frac{H(k)e^{-j\frac{\pi}{N}\left(k+\frac{1}{2}\right)}}{j\sin\left[\frac{\omega}{2}-\frac{\pi}{N}\left(k+\frac{1}{2}\right)\right]}$$



线性相位约束条件

第一种抽样方法				
	$h(n)$中心偶对称 N为奇数	$h(n)$中心偶对称 N为偶数	$h(n)$中心奇对称 N为奇数	$h(n)$中心奇对称 N为偶数
幅度约束	$\lvert H(k)\rvert=\lvert H(N-k)\rvert$ $k=0\sim(N-1)/2$	$\lvert H(k)\rvert=\lvert H(N-k)\rvert$ $k=0\sim(N/2-1)$ $\lvert H(N/2)\rvert=0$	$\lvert H(k)\rvert=\lvert H(N-k)\rvert$ $k=0\sim(N-1)/2$ $\lvert H(0)\rvert=0$	$\lvert H(k)\rvert=\lvert H(N-k)\rvert$ $k=0\sim(N/2-1)$ $\lvert H(0)\rvert=0$
相位约束	$\varphi(k)=-\varphi(N-k)$ $=-k(1-N^{-1})\pi$ $k=0\sim(N-1)/2$	$\varphi(k)=-\varphi(N-k)$ $=-k(1-N^{-1})\pi$ $k=0\sim(N/2-1)$	$\varphi(k)=-\varphi(N-k)$ $=\frac{\pi}{2}-k(1-N^{-1})\pi$ $k=0\sim(N-1)/2$	$\varphi(k)=-\varphi(N-k)$ $=\frac{\pi}{2}-k(1-N^{-1})\pi$ $k=0\sim(N/2-1)$ $\varphi(N/2)=0$

对于第一种抽样方式，当h(n)为实数时

$$h(n) = h^*(n)$$

$$H(k) = DFT[h(n)]$$

根据P91 （3-79）

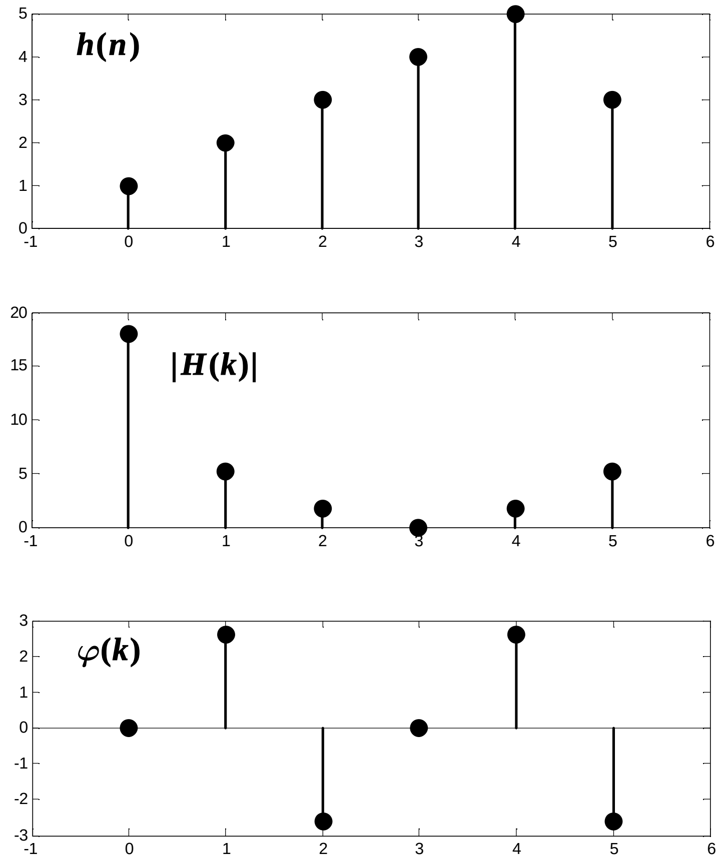
$$H^*(N-k) = DFT[h^*(n)]$$

于是

$$H(k) = H^*(N-k)$$

$$\begin{cases} |H(k)| = |H(N-k)| \\ \theta(k) = \arg[H(k)] = -\theta(N-k) \end{cases}$$

以$k = \dfrac{N}{2}$中心

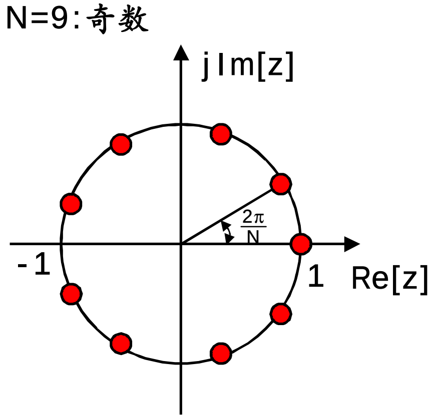


线性相位系统传函和频响

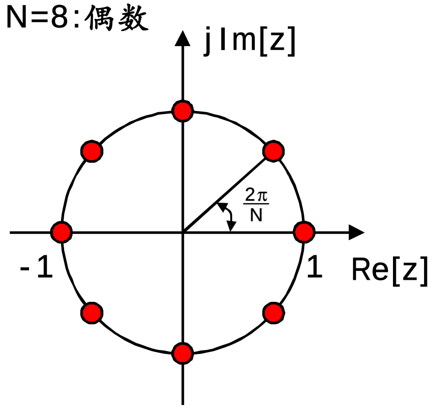
第一种频率抽样方法：

由： $\theta(\omega)=-\left(\frac{N-1}{2}\right)\omega$

N为奇数： $\theta(k)=\begin{cases}-\frac{2\pi}{N}k\left(\frac{N-1}{2}\right) & k=0,\ldots,\frac{N-1}{2}\\ \frac{2\pi}{N}(N-k)\left(\frac{N-1}{2}\right) & k=\frac{N+1}{2},\ldots,N-1\end{cases}$



N为偶数： $\theta(k)=\begin{cases}-\frac{2\pi}{N}k\left(\frac{N-1}{2}\right) & k=0,\ldots,\left(\frac{N}{2}-1\right)\\ 0 & k=\frac{N}{2}\\ \frac{2\pi}{N}(N-k)\left(\frac{N-1}{2}\right) & k=\left(\frac{N}{2}+1\right),\ldots,N-1\end{cases}$



当N为奇数时：

$$H(k)=\begin{cases}\left|H(k)\right|e^{-j\frac{2\pi}{N}k\left(\frac{N-1}{2}\right)} & k=0,\dots,\dfrac{N-1}{2}\\ \left|H(k)\right|e^{j\frac{2\pi}{N}(N-k)\left(\frac{N-1}{2}\right)} & k=\dfrac{N+1}{2},\dots,N-1\end{cases}$$

当N为偶数时：

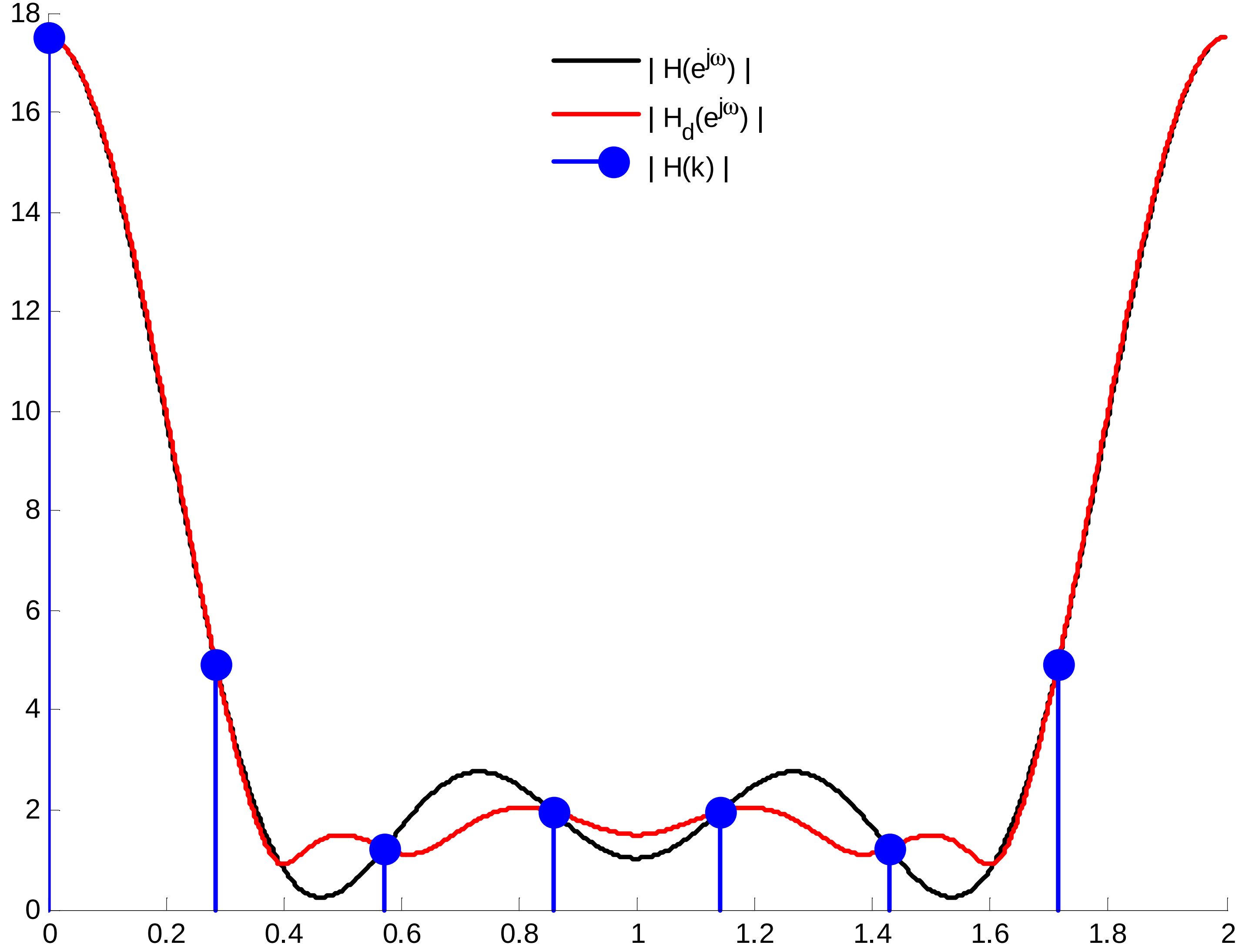
$$H(k)=\begin{cases}\left|H(k)\right|e^{-j\frac{2\pi}{N}k\left(\frac{N-1}{2}\right)} & k=0,\dots,\left(\dfrac{N}{2}-1\right)\\ 0 & k=\dfrac{N}{2}\\ \left|H(k)\right|e^{j\frac{2\pi}{N}(N-k)\left(\frac{N-1}{2}\right)} & k=\left(\dfrac{N}{2}+1\right),\dots,N-1\end{cases}$$

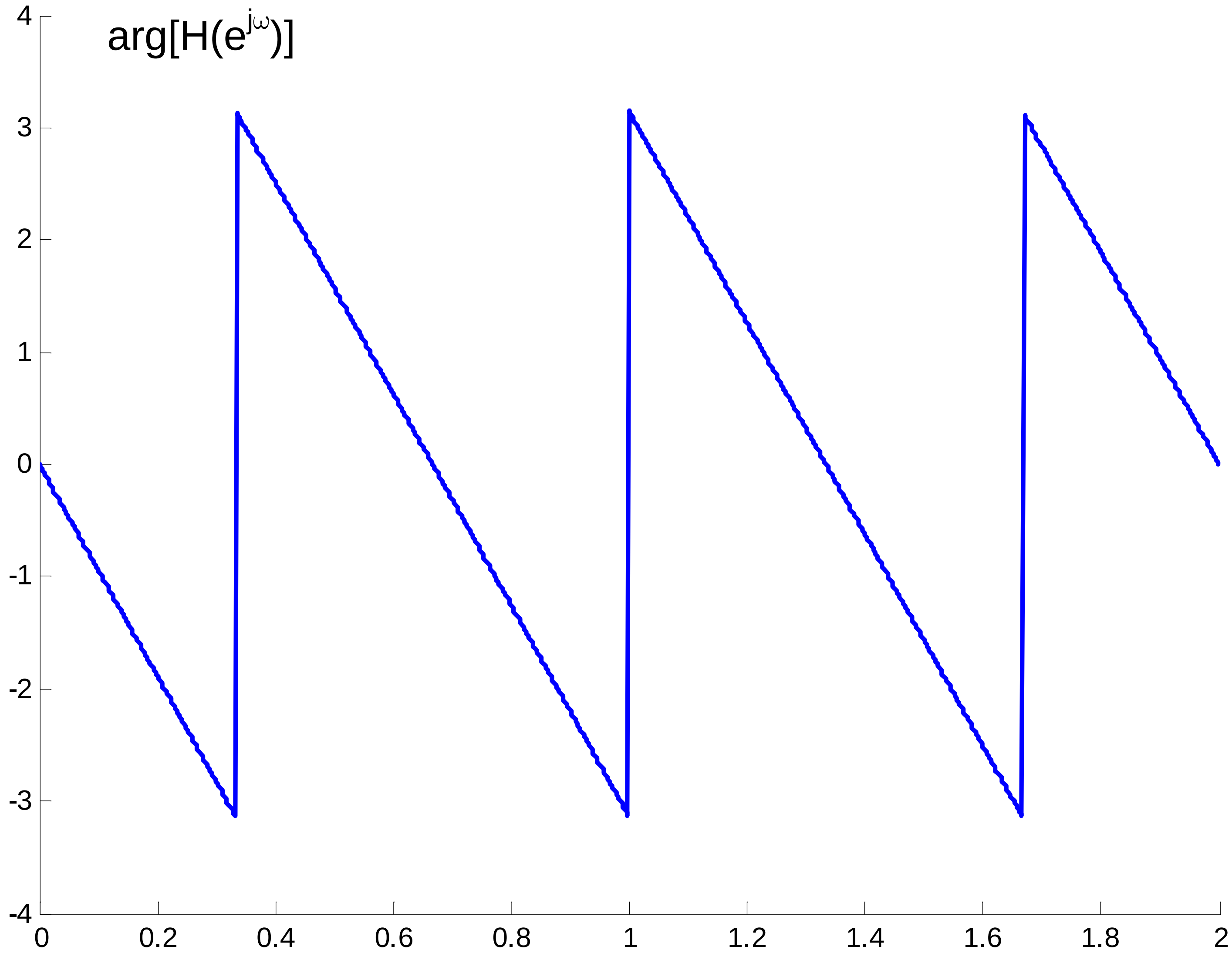
频率响应：

$$H\left(e^{j\omega}\right)=e^{-j\left(\frac{N-1}{2}\right)\omega}\left\{\frac{\left|H(0)\right|\sin\left(\frac{\omega N}{2}\right)}{N\sin\left(\frac{\omega}{2}\right)}+\sum_{k=1}^{M}\frac{|H(k)|}{N}\left[\frac{\sin\left[N\left(\frac{\omega}{2}-\frac{k\pi}{N}\right)\right]}{\sin\left(\frac{\omega}{2}-\frac{k\pi}{N}\right)}+\frac{\sin\left[N\left(\frac{\omega}{2}+\frac{k\pi}{N}\right)\right]}{\sin\left(\frac{\omega}{2}+\frac{k\pi}{N}\right)}\right]\right\}$$

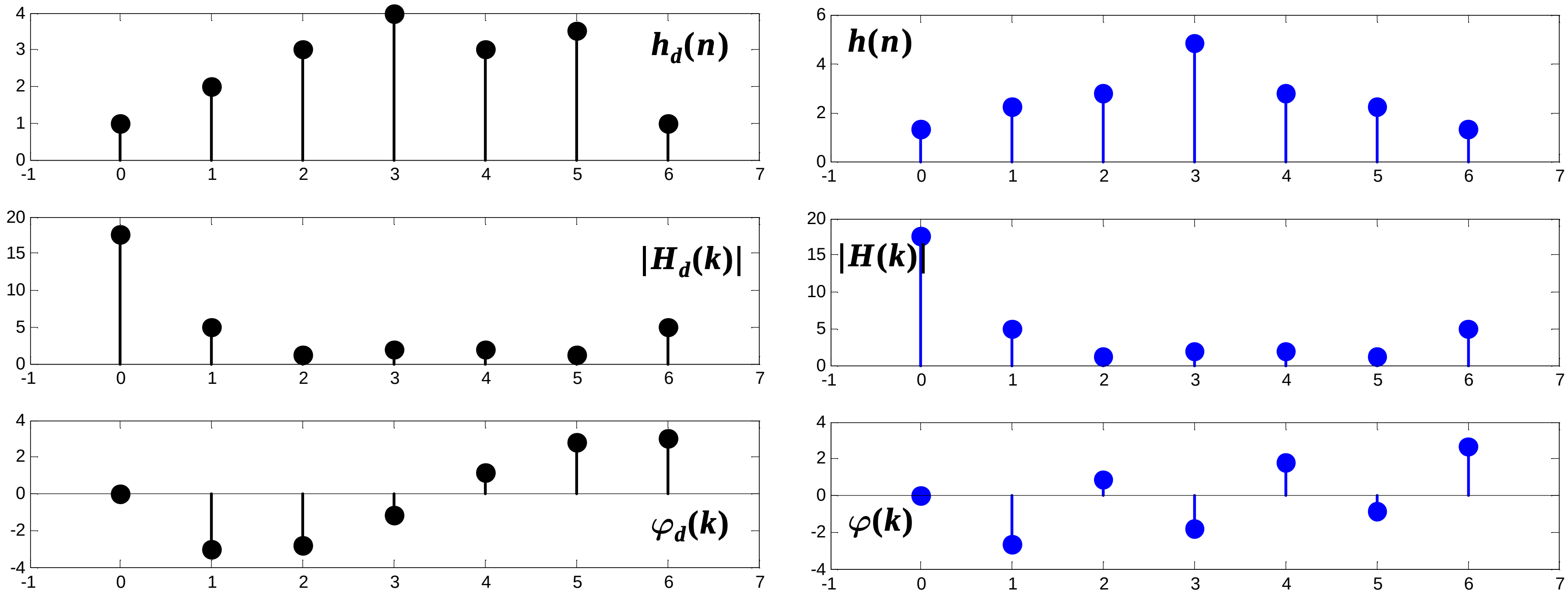
N为奇数$M=(N-1)/2$，　N为偶数$M=N/2-1$

第一种抽样
偶对称　$N = 7$
奇数点



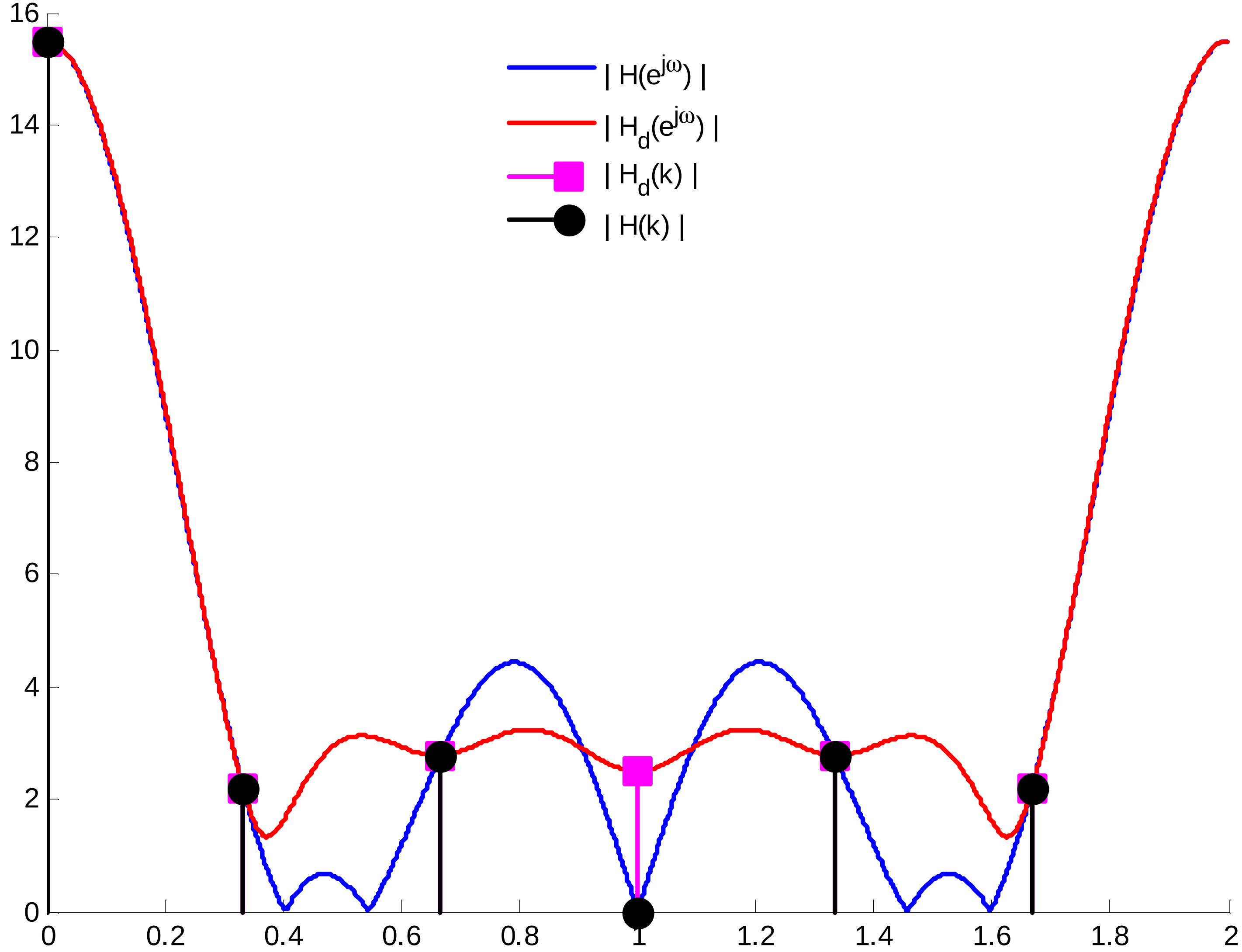


arg[H(e^jω)]
4
3
2
1
0
-1
-2
-3
-4
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

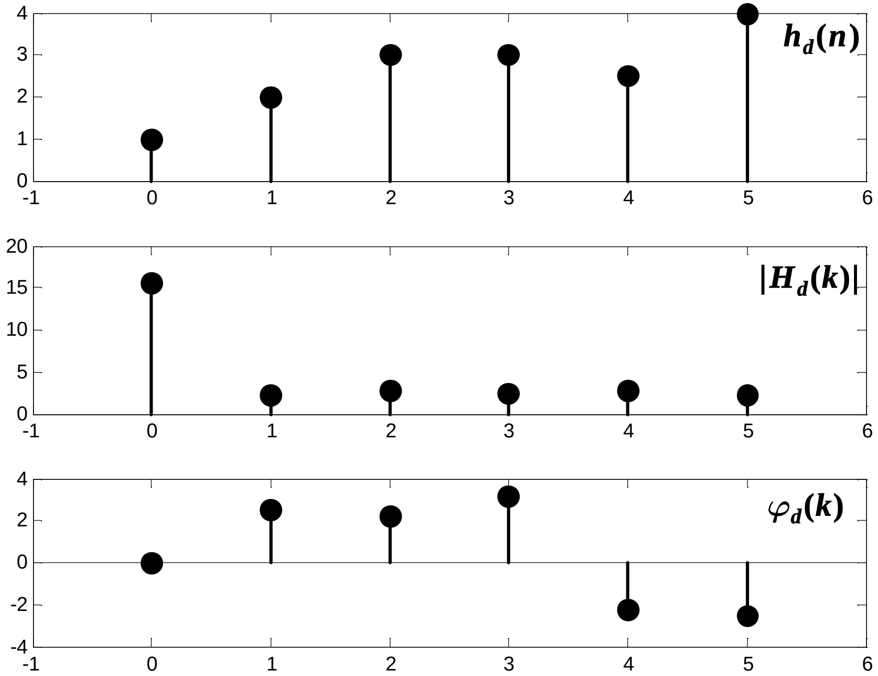


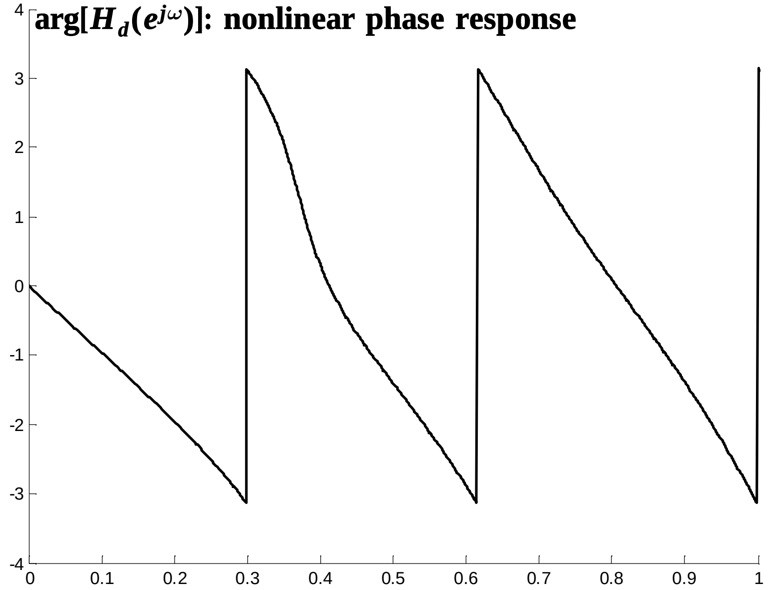
$h_d(n)$
$h(n)$
$|H_d(k)|$
$|H(k)|$
$\varphi_d(k)$
$\varphi(k)$

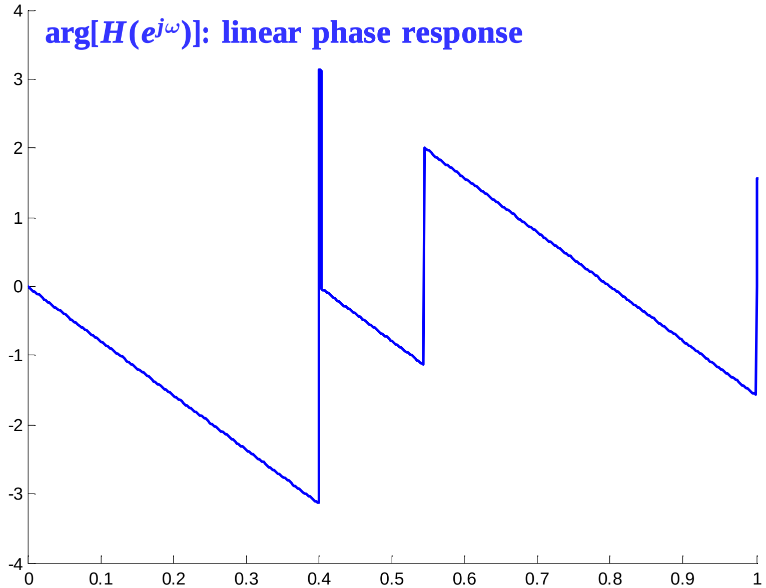
第一种抽样
偶对称 $N = 6$
偶数点

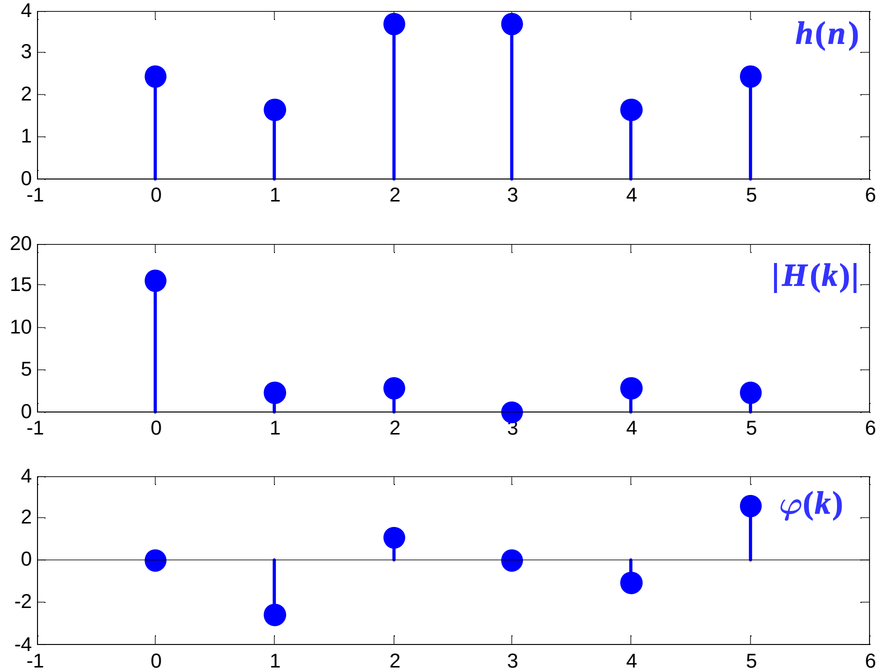


强行置零

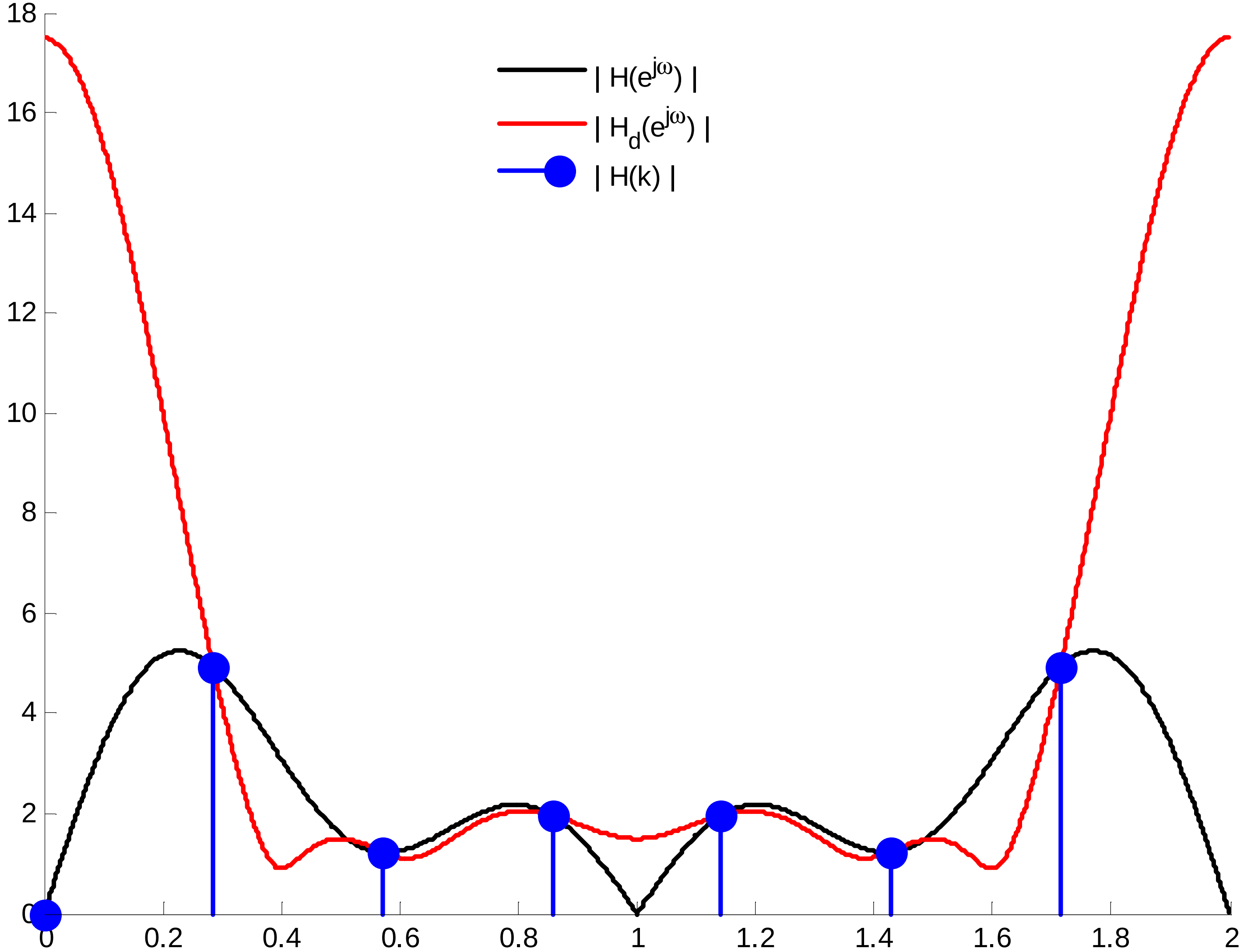

$h_d(n)$
$|H_d(k)|$
$\varphi_d(k)$

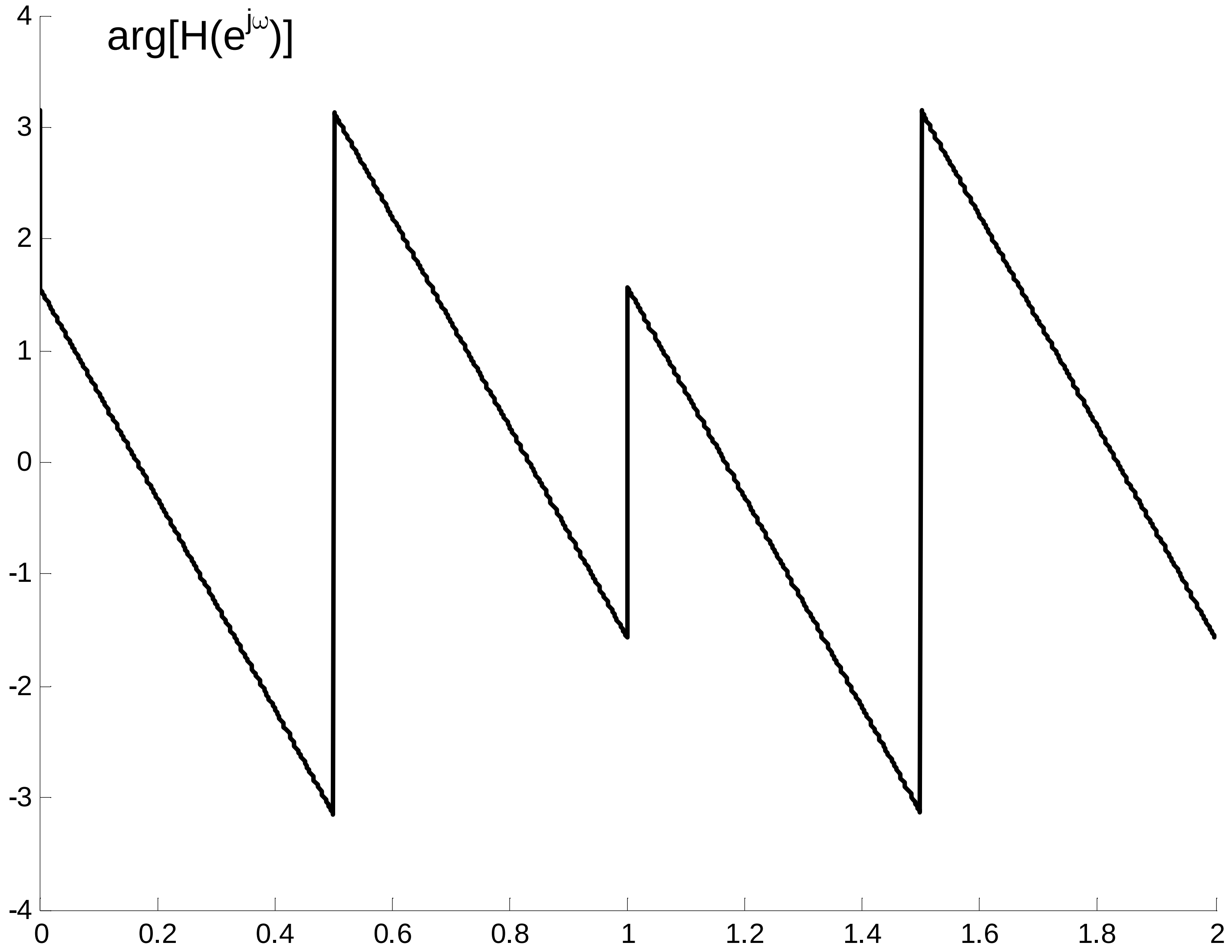

$\arg[H_d(e^{j\omega})]$: nonlinear phase response


$\arg[H(e^{j\omega})]$: linear phase response

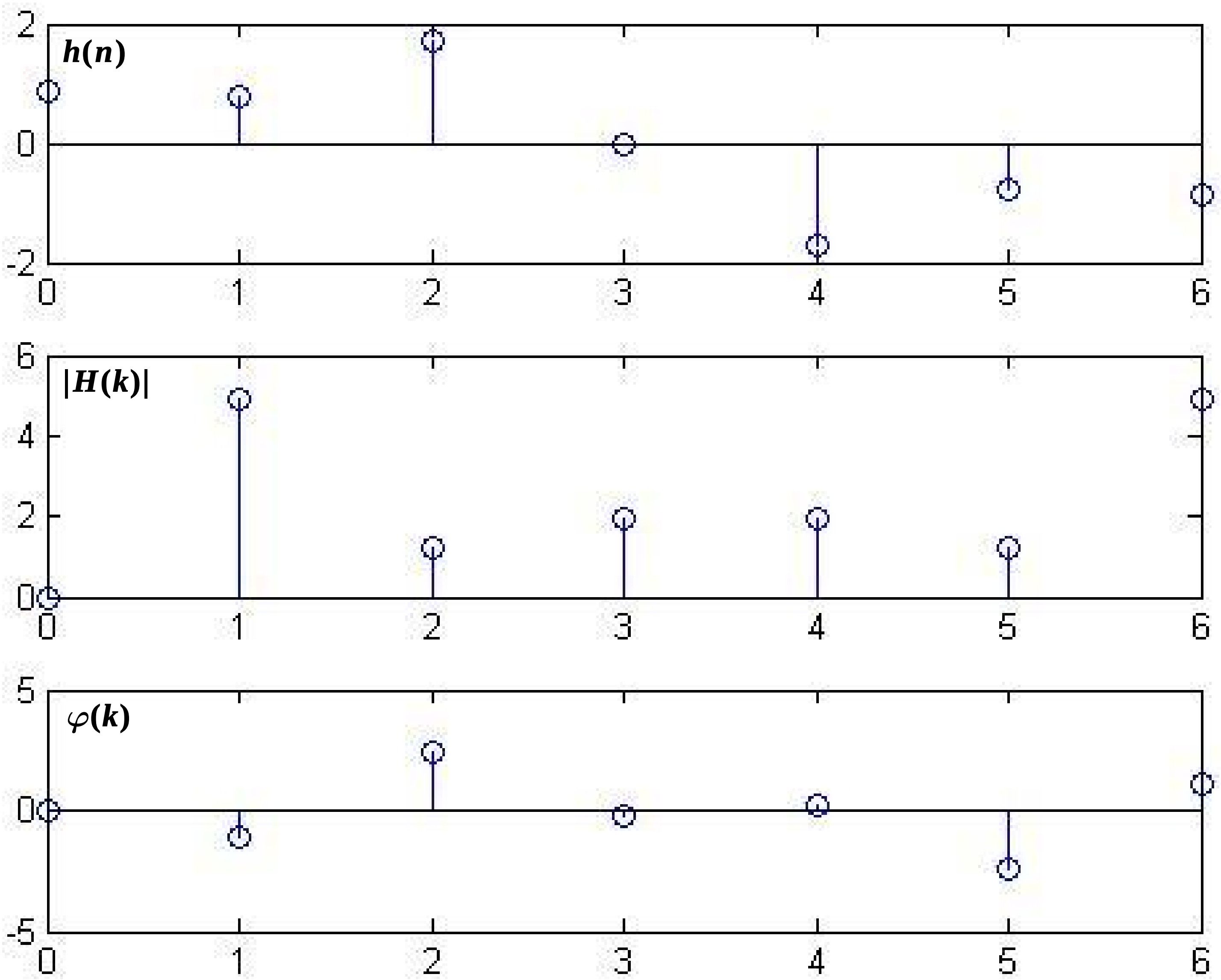

$h(n)$
$|H(k)|$
$\varphi(k)$

第一种抽样
奇对称 $N = 7$
奇数点



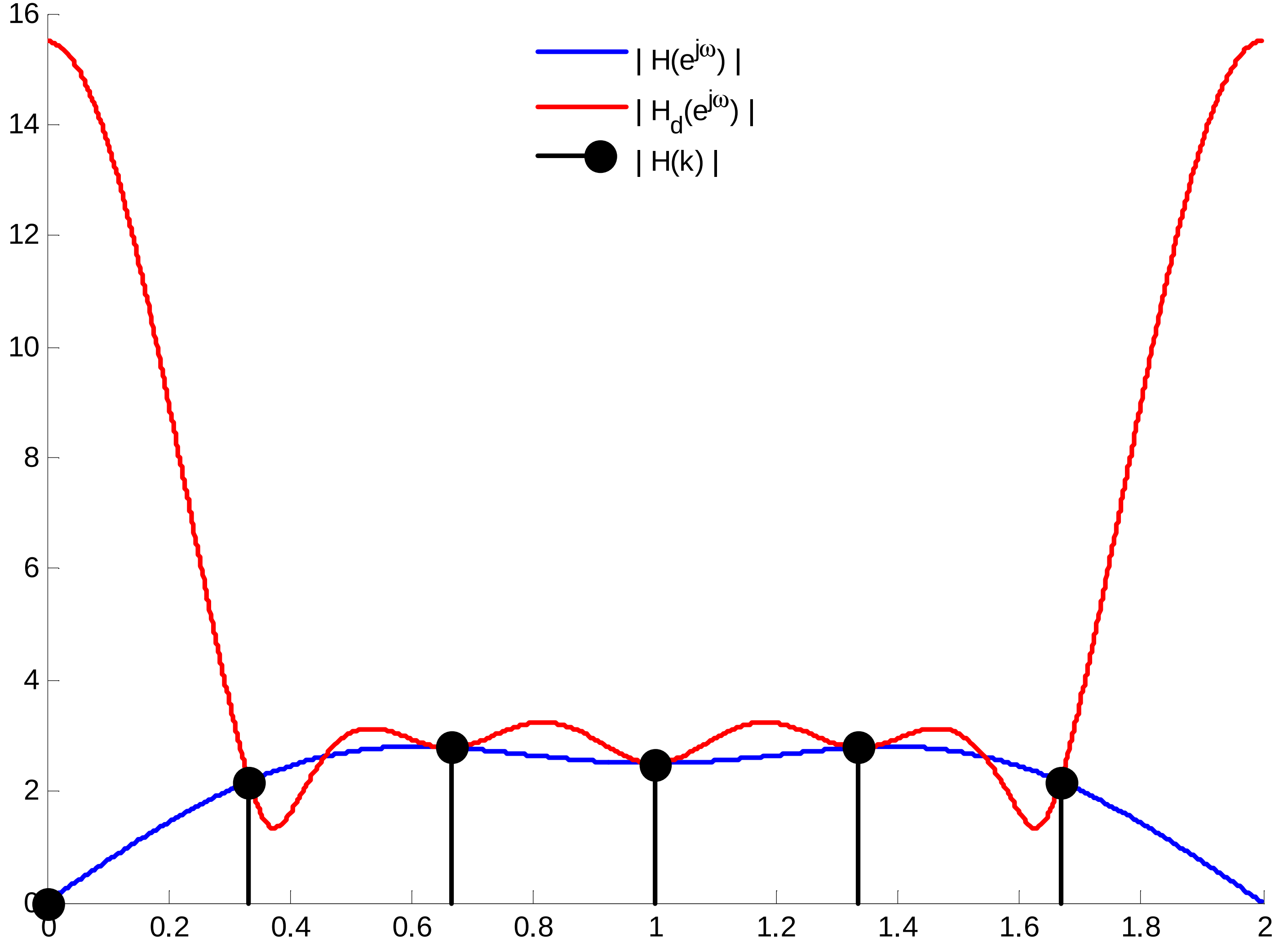


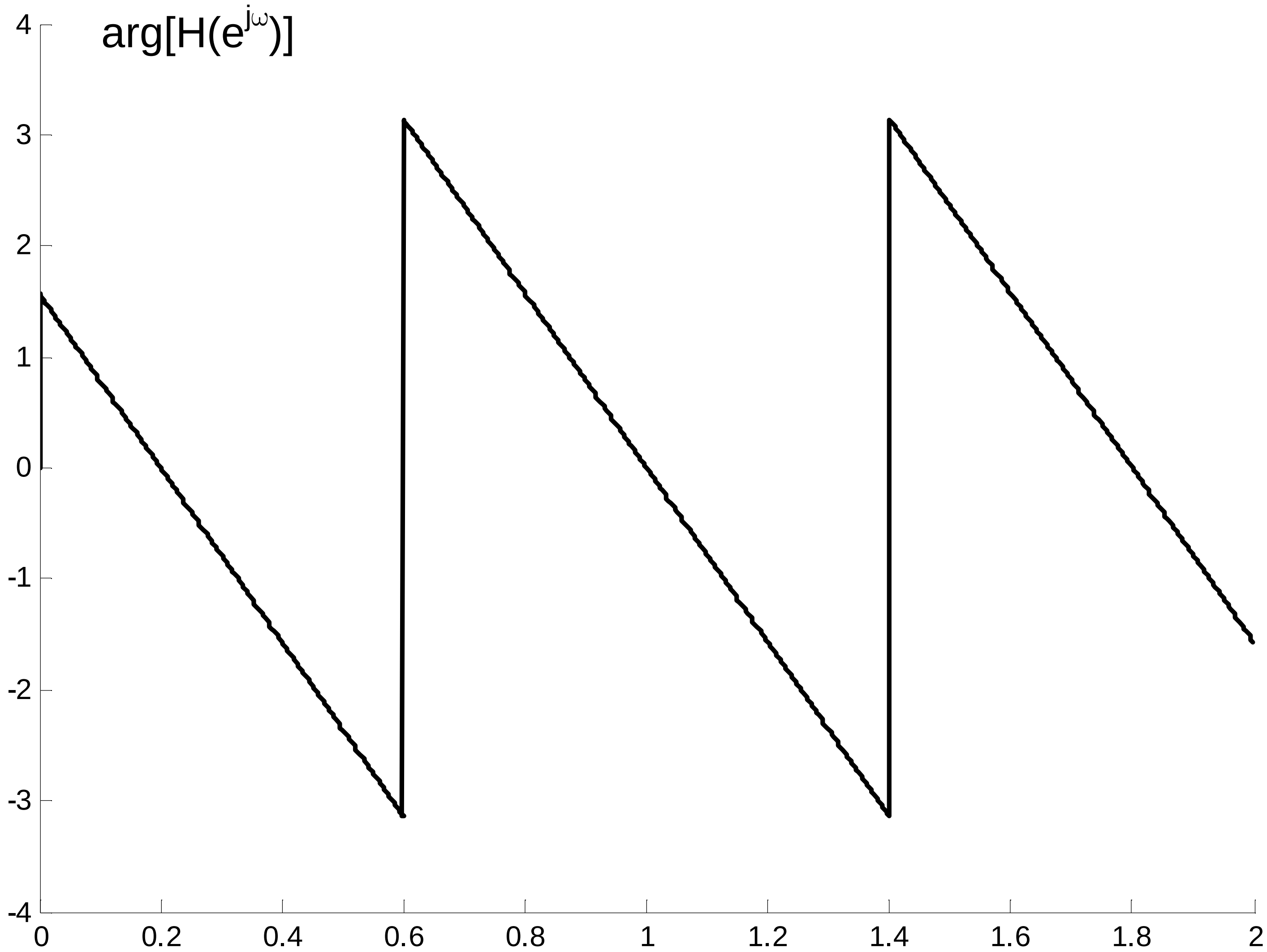
arg[H(e^jω)]
4
3
2
1
0
-1
-2
-3
-4
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2



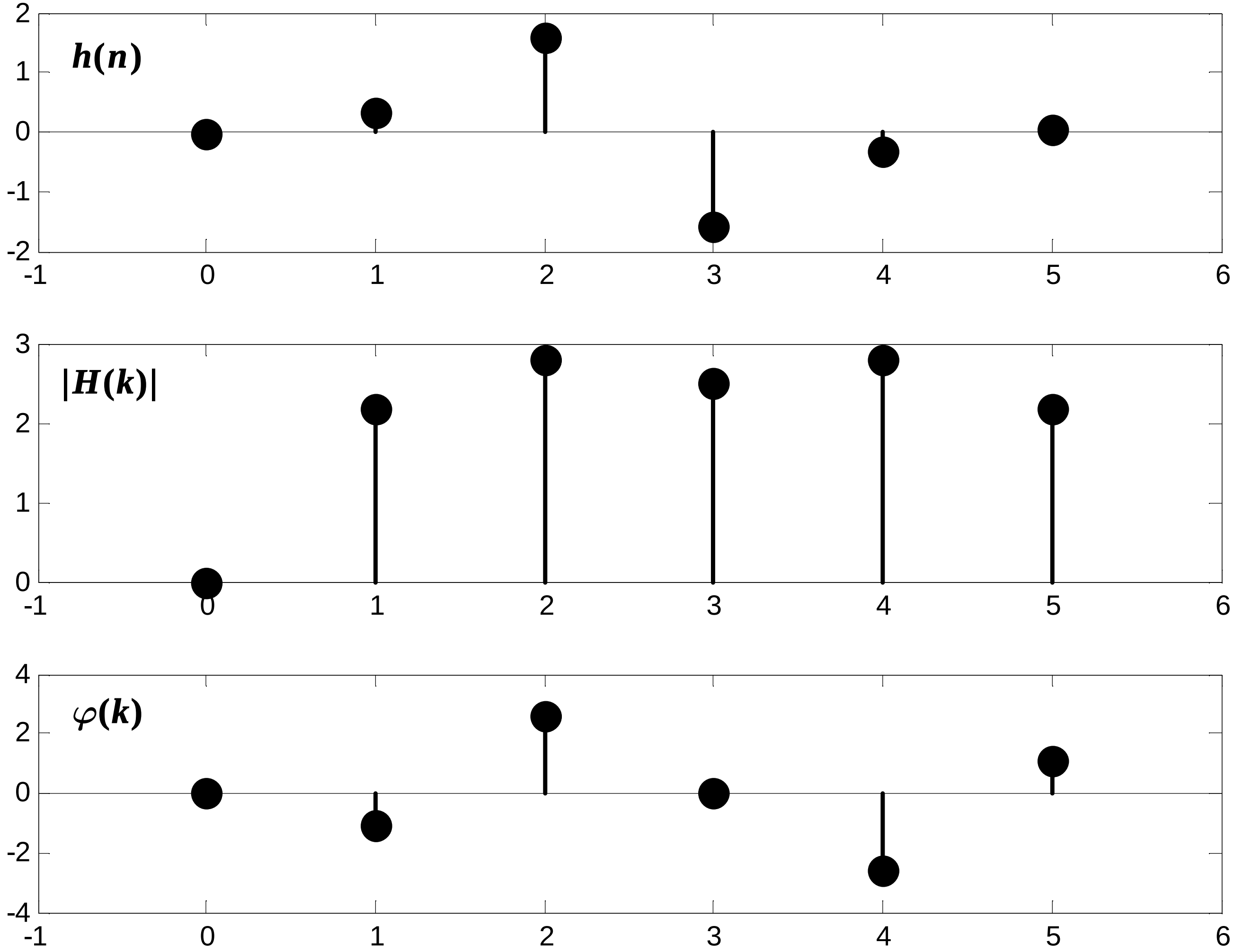
h(n)
|H(k)|
φ(k)
2
0
-2
6
4
2
0
5
0
-5
0
1
2
3
4
5
6

第一种抽样
奇对称　$N = 6$
偶数点





arg[H(e^jω)]
4
3
2
1
0
-1
-2
-3
-4
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2



$h(n)$
$|H(k)|$
$\varphi(k)$
2
1
0
-1
-2
3
4
-4
-1
0
1
2
3
4
5
6

线性相位约束条件

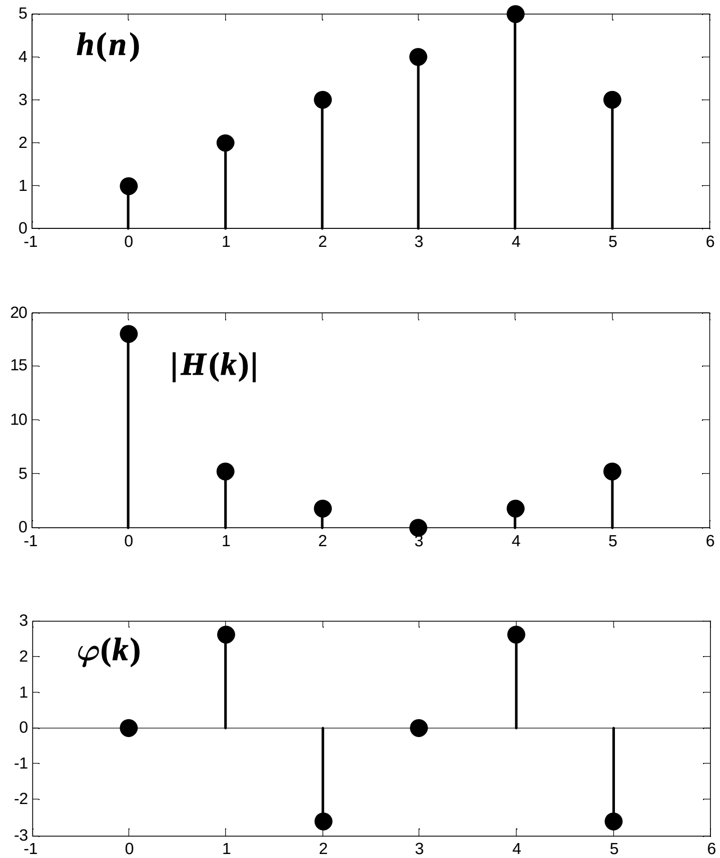
第二种抽样方法				
	$h(n)$ 中心偶对称 N 为奇数	$h(n)$ 中心偶对称 N 为偶数	$h(n)$ 中心奇对称 N 为奇数	$h(n)$ 中心奇对称 N 为偶数
幅度约束	$\lvert H(k)\rvert=\lvert H(N-k-1)\rvert$ $k=0\sim(N-1)/2$	$\lvert H(k)\rvert=\lvert H(N-k-1)\rvert$ $k=0\sim(N/2-1)$	$\lvert H(k)\rvert=\lvert H(N-k-1)\rvert$ $k=0\sim(N-1)/2$ $\lvert H(\frac{N-1}{2})\rvert=0$	$\lvert H(k)\rvert=\lvert H(N-k-1)\rvert$ $k=0\sim(N/2-1)$
相位约束	$\varphi(k)=-\varphi(N-k-1)$ $=-(k+\frac{1}{2})(1-\frac{1}{N})\pi$ $k=0\sim(N-3)/2$ $\varphi(\frac{N-1}{2})=0$	$\varphi(k)=-\varphi(N-k-1)$ $=-(k+\frac{1}{2})(1-\frac{1}{N})\pi$ $k=0\sim(N/2-1)$	$\varphi(k)=-\varphi(N-k-1)$ $=\frac{\pi}{2}-(k+\frac{1}{2})(1-\frac{1}{N})\pi$ $k=0\sim(N-3)/2$	$\varphi(k)=-\varphi(N-k-1)$ $=\frac{\pi}{2}-(k+\frac{1}{2})(1-\frac{1}{N})\pi$ $k=0\sim(N/2-1)$

对于第二种抽样方式，当h(n)为实数时

$$H(k) = H^*(N-1-k)$$

$$\begin{cases} |H(k)| = |H(N-1-k)| \\ \theta(k) = \arg[H(k)] = -\theta(N-1-k) \end{cases}$$

以$k = \dfrac{N-1}{2}$中心

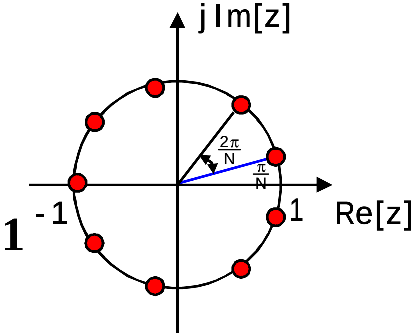


线性相位系统传函和频响

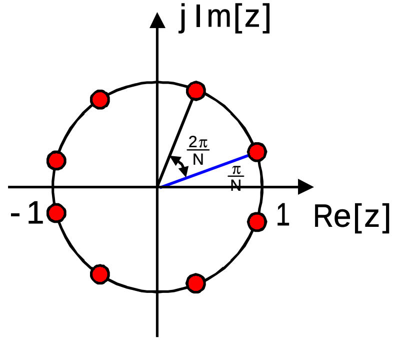
第二种频率抽样方法：

$$N\text{为奇数：}\ \theta(k)=\begin{cases}-\dfrac{2\pi}{N}\left(k+\dfrac{1}{2}\right)\left(\dfrac{N-1}{2}\right) & k=0,\ldots,\dfrac{N-3}{2}\\ 0 & k=\dfrac{N-1}{2}\\ \dfrac{2\pi}{N}\left(N-k-\dfrac{1}{2}\right)\left(\dfrac{N-1}{2}\right) & k=\dfrac{N+1}{2},\ldots,N-1\end{cases}$$

N=9:奇数



N=8:偶数



$$N\text{为偶数：}\ \theta(k)=\begin{cases}-\dfrac{2\pi}{N}\left(k+\dfrac{1}{2}\right)\left(\dfrac{N-1}{2}\right) & k=0,\ldots,\left(\dfrac{N}{2}-1\right)\\ \dfrac{2\pi}{N}\left(N-k-\dfrac{1}{2}\right)\left(\dfrac{N-1}{2}\right) & k=\dfrac{N}{2},\ldots,N-1\end{cases}$$

当N为奇数时：

$$H(k)=\begin{cases}|H(k)|e^{-j\frac{2\pi}{N}\left(k+\frac{1}{2}\right)\left(\frac{N-1}{2}\right)} & k=0,\dots,\frac{N-3}{2}\\ \left|H\left(\frac{N-1}{2}\right)\right| & k=\frac{N-1}{2}\\ |H(k)|e^{j\frac{2\pi}{N}\left(N-k-\frac{1}{2}\right)\left(\frac{N-1}{2}\right)} & k=\frac{N+1}{2},\dots,N-1\end{cases}$$

当N为偶数时：

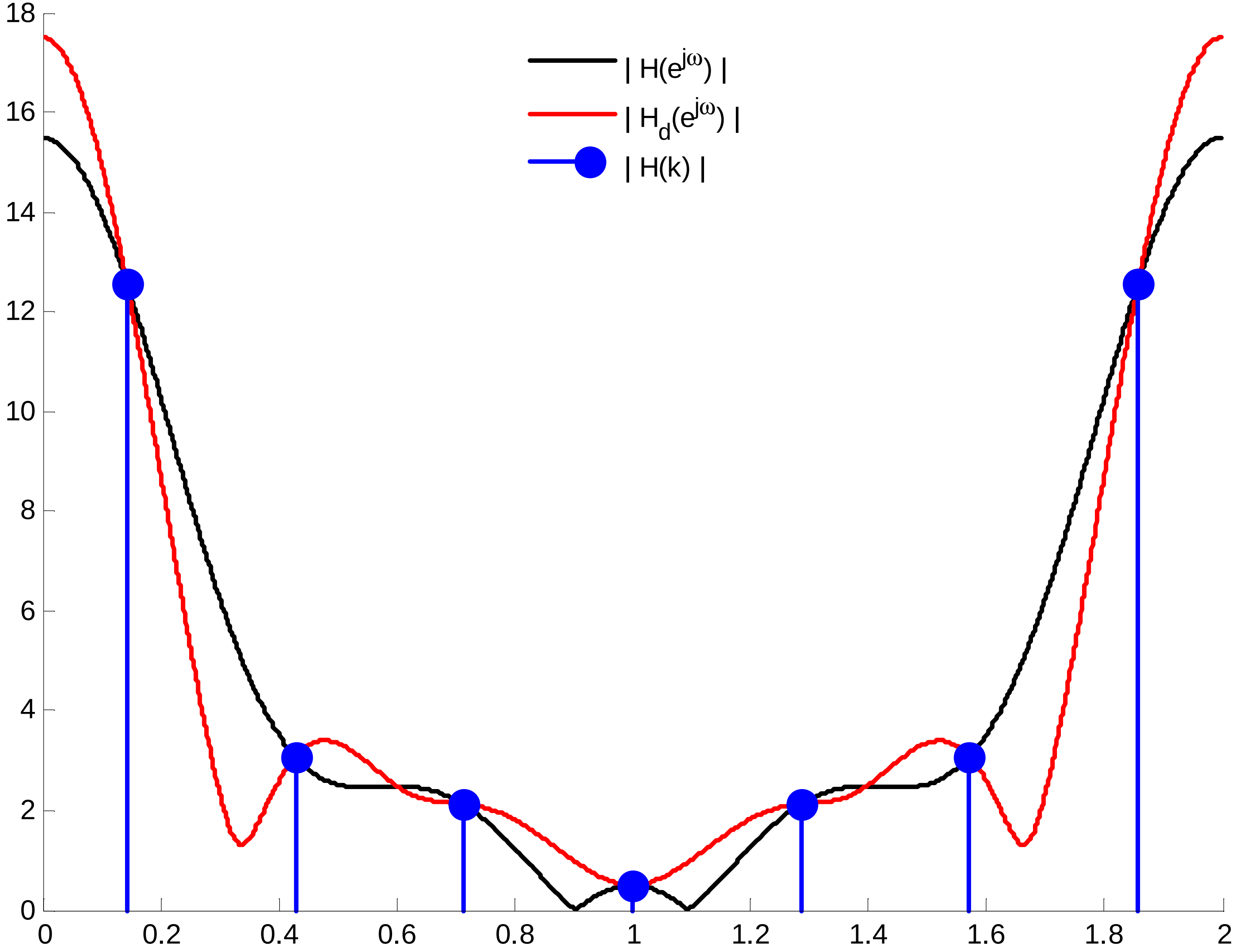
$$H(k)=\begin{cases}|H(k)|e^{-j\frac{2\pi}{N}\left(k+\frac{1}{2}\right)\left(\frac{N-1}{2}\right)} & k=0,\dots,\left(\frac{N}{2}-1\right)\\ |H(k)|e^{j\frac{2\pi}{N}\left(N-k-\frac{1}{2}\right)\left(\frac{N-1}{2}\right)} & k=\frac{N}{2},\dots,N-1\end{cases}$$

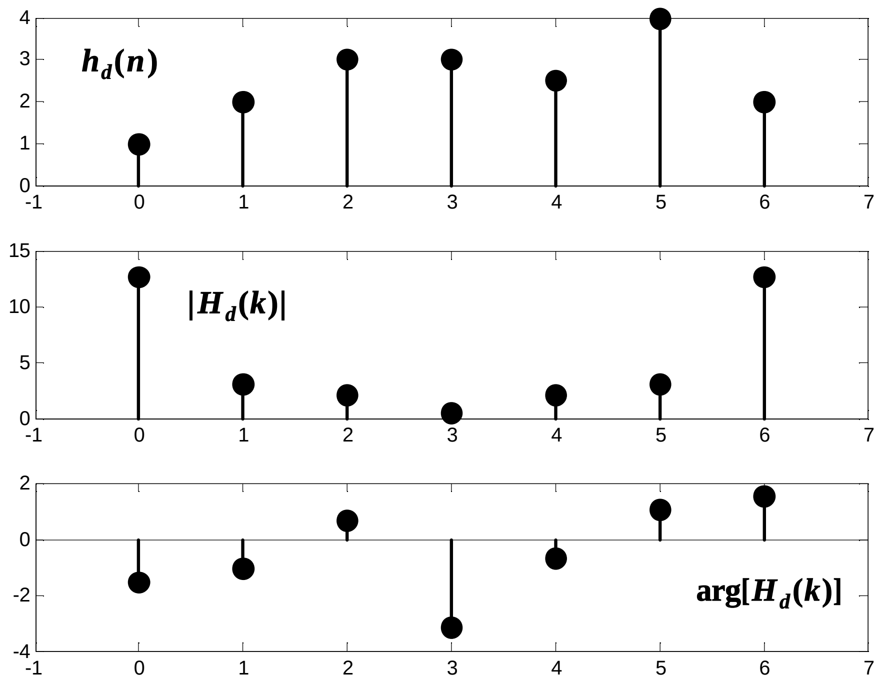
频率响应：

$$H\left(e^{j\omega}\right)=e^{-j\left(\frac{N-1}{2}\right)\omega}\left\{H_{\frac{N-1}{2}}(\omega)+\sum_{k=0}^{M}\frac{|H(k)|}{N}\left[\frac{\sin\left\{N\left[\frac{\omega}{2}-\frac{\pi}{N}\left(k+\frac{1}{2}\right)\right]\right\}}{\sin\left[\frac{\omega}{2}-\frac{\pi}{N}\left(k+\frac{1}{2}\right)\right]}+\frac{\sin\left\{N\left[\frac{\omega}{2}+\frac{\pi}{N}\left(k+\frac{1}{2}\right)\right]\right\}}{\sin\left[\frac{\omega}{2}+\frac{\pi}{N}\left(k+\frac{1}{2}\right)\right]}\right]\right\}$$

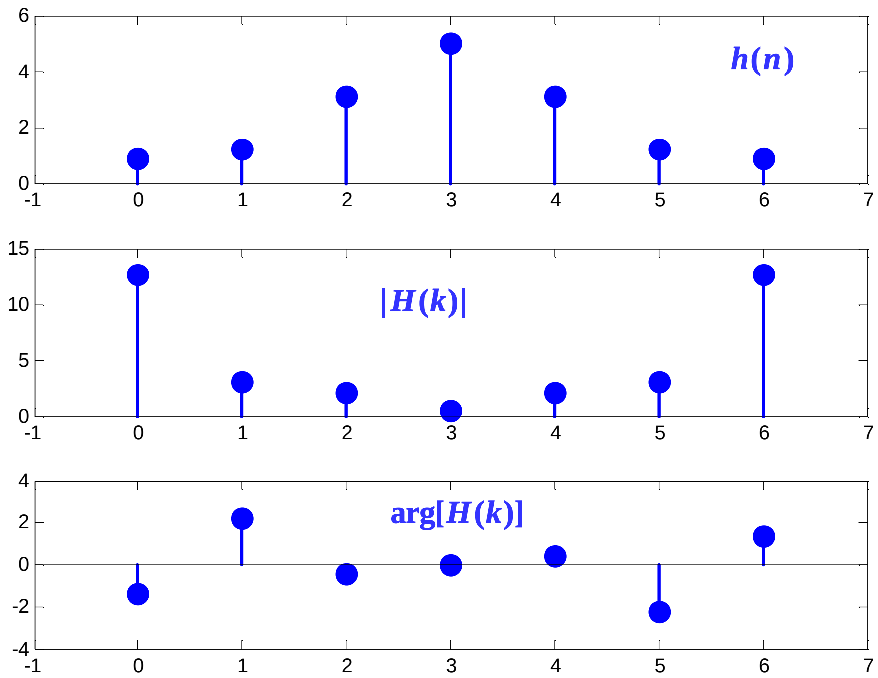
$$\begin{cases}H_{\frac{N-1}{2}}(\omega)=\frac{\left|H\left(\frac{N-1}{2}\right)\right|}{N}\cdot\frac{\cos\left(\frac{\omega N}{2}\right)}{\cos\left(\frac{\omega}{2}\right)},\ M=\frac{N-3}{2} & n:odd\\ H_{\frac{N-1}{2}}(\omega)=0\qquad ,M=\frac{N}{2}-1 & n:even\end{cases}$$

第二种抽样
偶对称 $N = 7$
奇数点

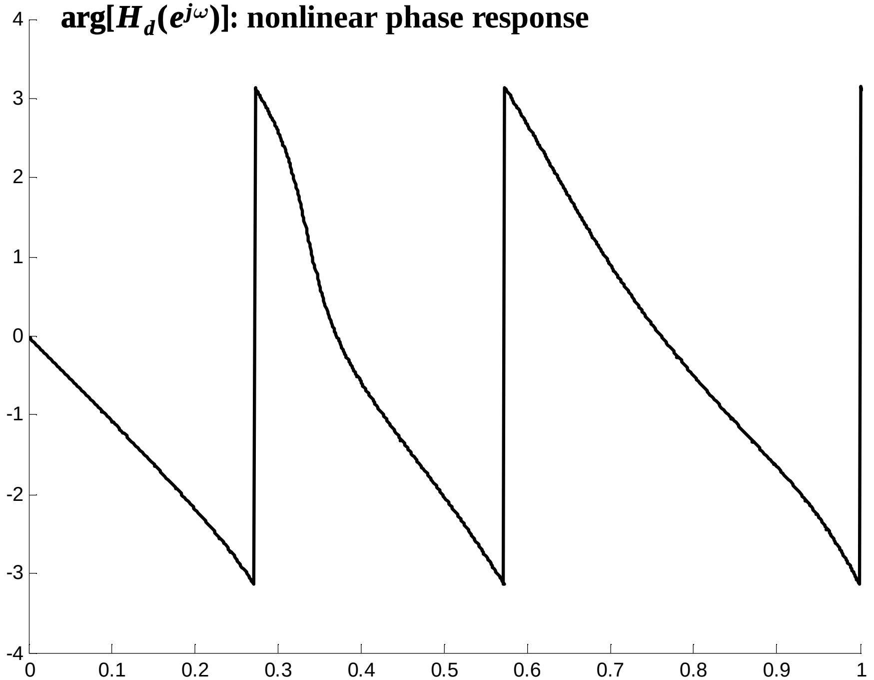


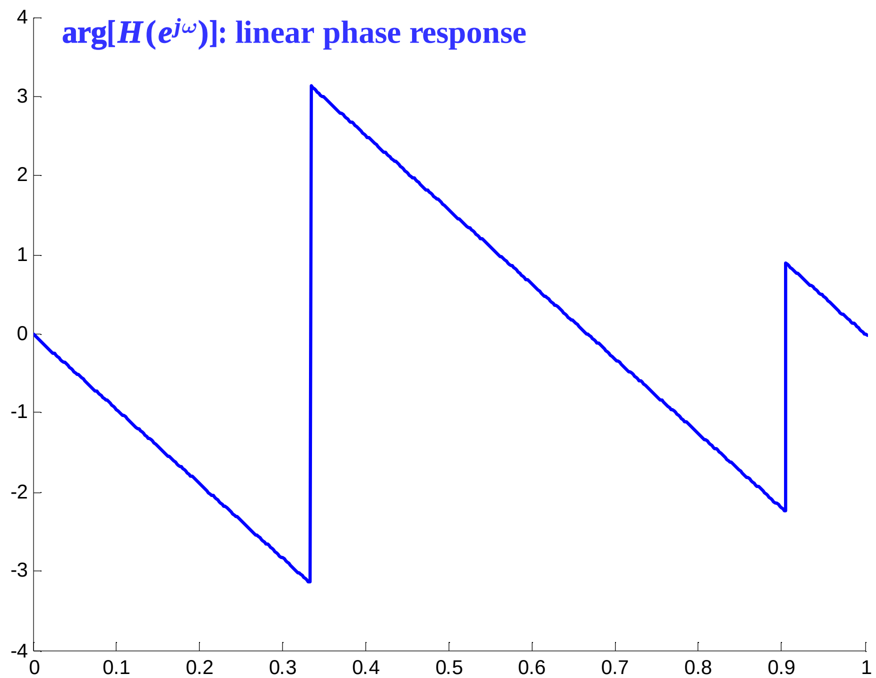


$h_d(n)$
$|H_d(k)|$
$\arg[H_d(k)]$



$h(n)$
$|H(k)|$
$\arg[H(k)]$

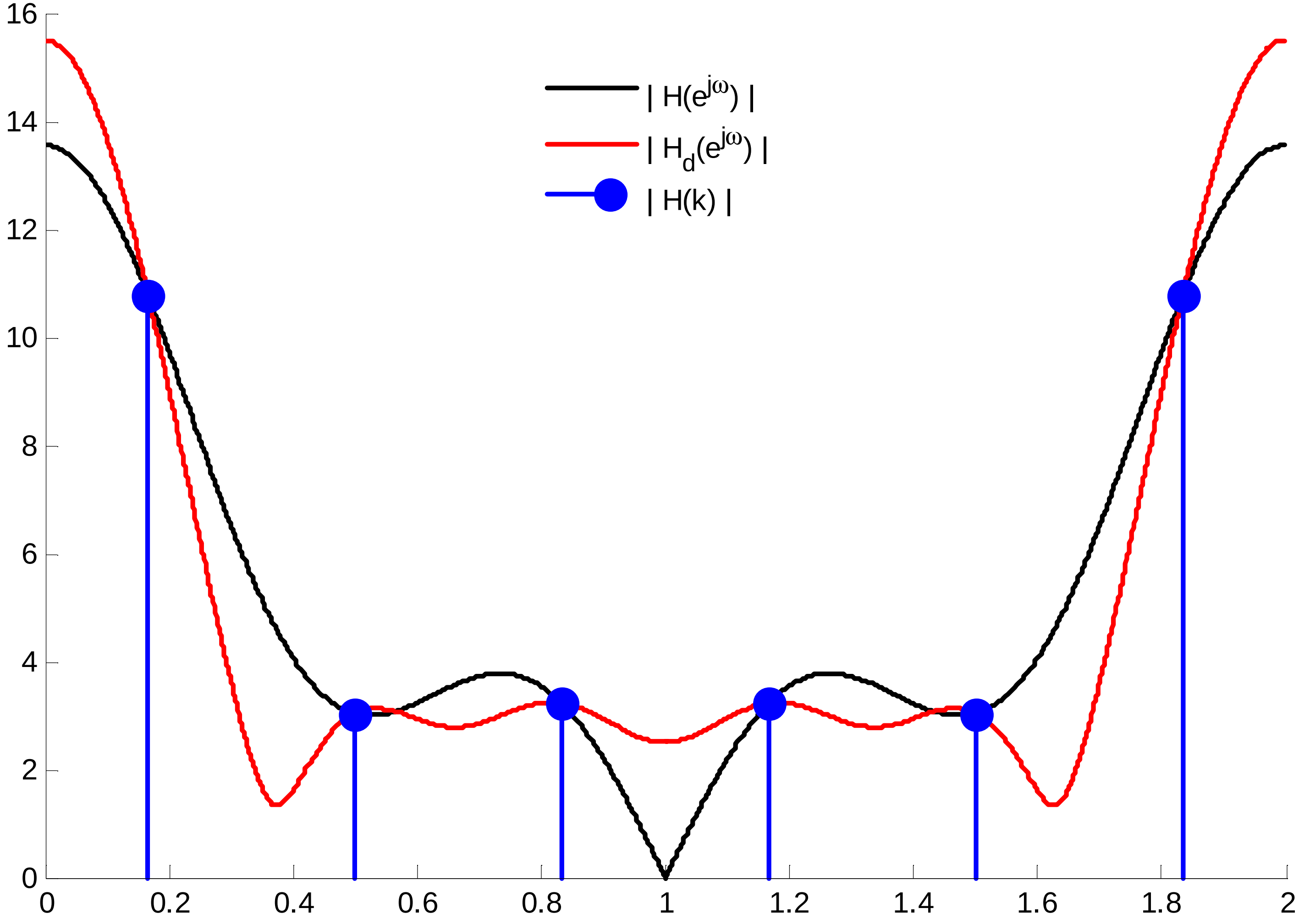

arg[$H_d(e^{j\omega})$]: nonlinear phase response
4
3
2
1
0
-1
-2
-3
-4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

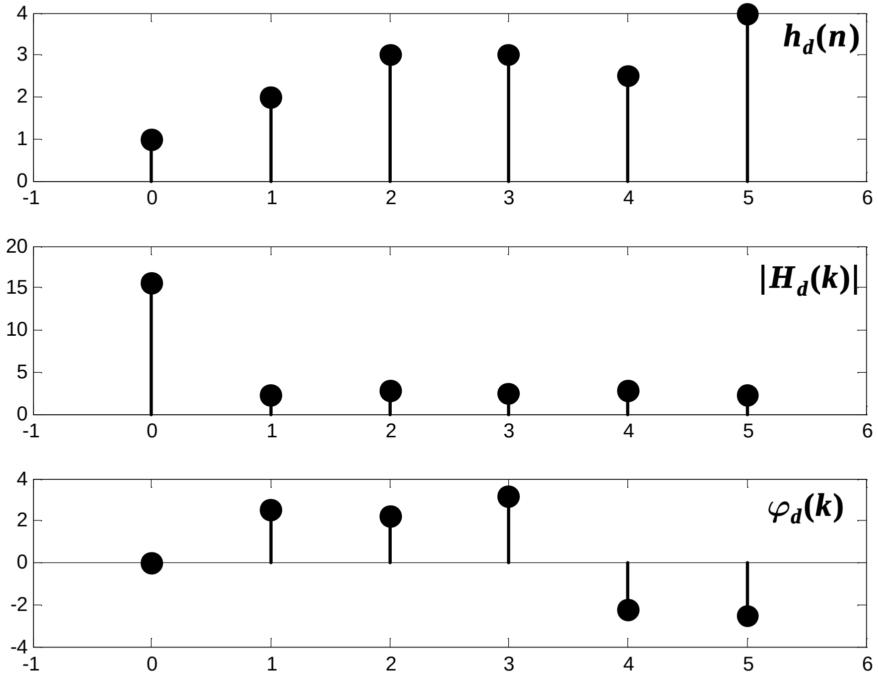

arg[$H(e^{j\omega})$]: linear phase response
4
3
2
1
0
-1
-2
-3
-4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

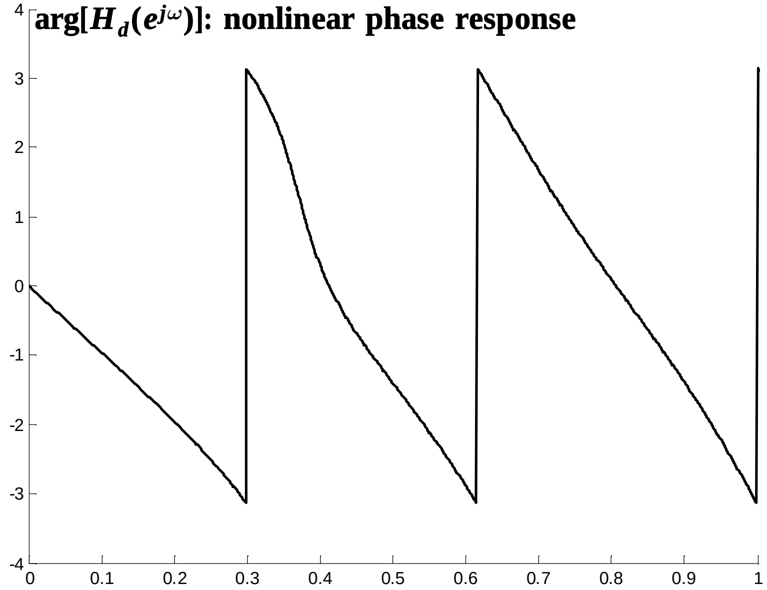
第二种抽样

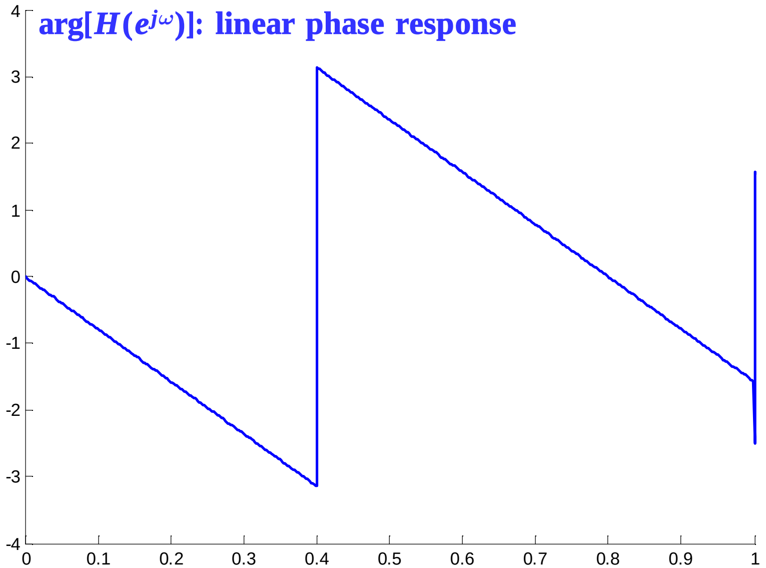
偶对称 $N = 6$

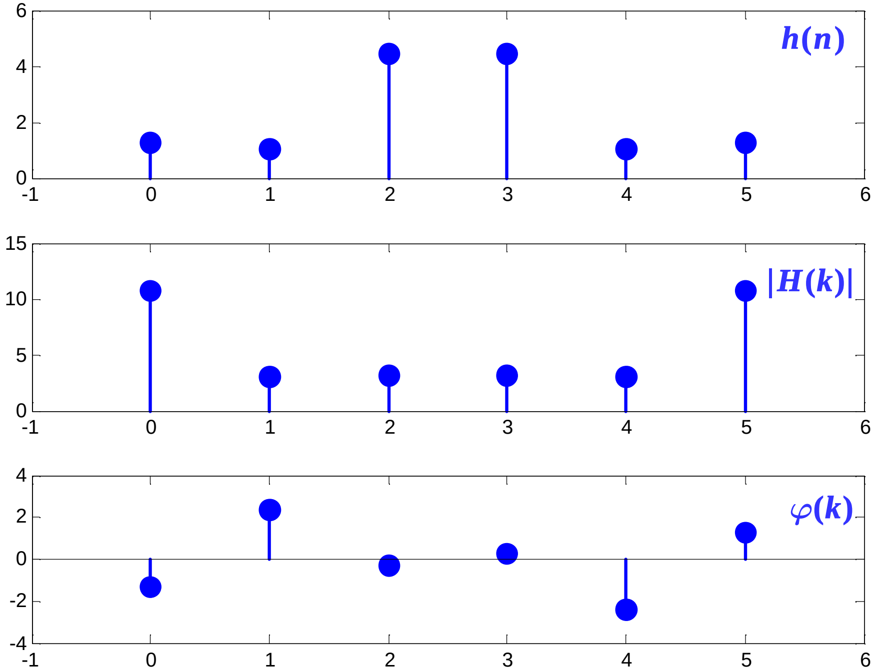
偶数点



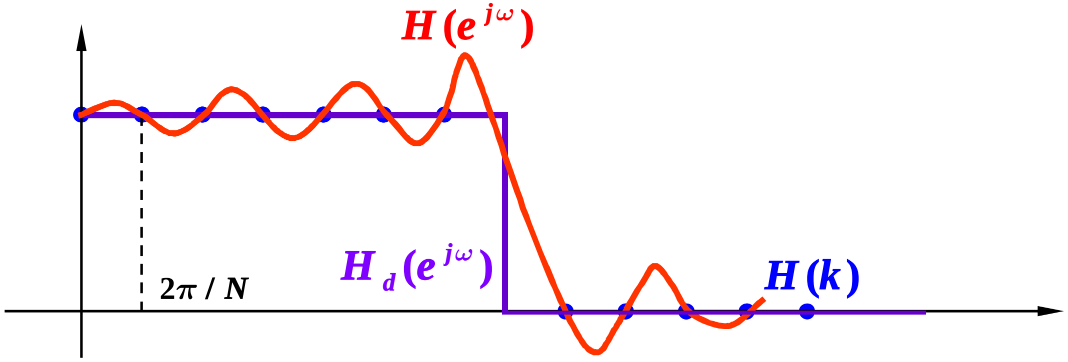

$h_d(n)$
$|H_d(k)|$
$\varphi_d(k)$


arg[$H_d(e^{j\omega})$]: nonlinear phase response


arg[$H(e^{j\omega})$]: linear phase response


$h(n)$
$|H(k)|$
$\varphi(k)$

过渡带的优化设计（增加自由度）



增加过渡带抽样点，可加大阻带衰减 ξ

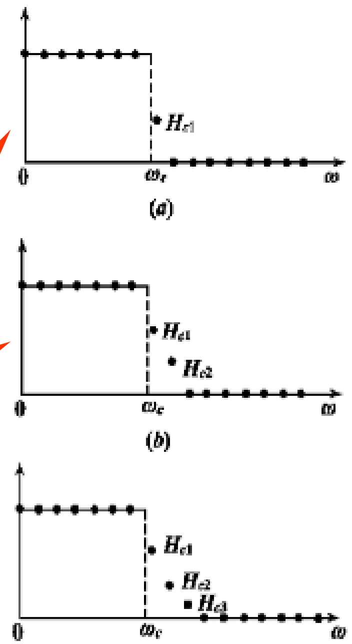
$$H(e^{j\omega}) = \sum_{k=0}^{N-1} H(k)\Phi\left(\omega - \frac{2\pi}{N}k\right)$$

不加过渡抽样点： $\xi = -20\text{dB}$

加一点： $\xi = -40 \sim -54\text{dB}$

加两点： $\xi = -60 \sim -75\text{dB}$

加三点： $\xi = -80 \sim -95\text{dB}$



NOTE

- 增加过渡带抽样点，可加大阻带衰减，但导致过渡带变宽
- 增加N，使抽样点变密，减小过渡带宽度，但增加了计算量

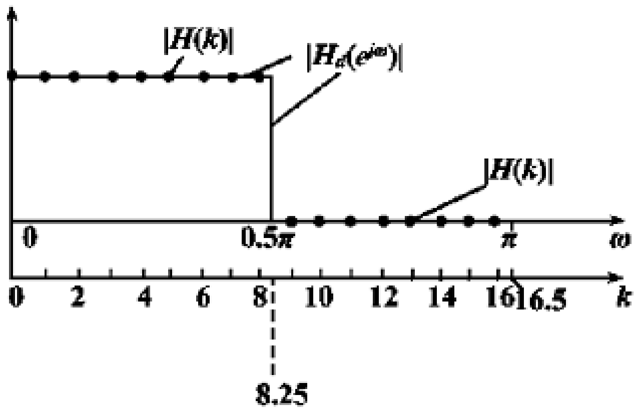
优点： 频域直接设计；窄带

缺点： 抽样频率只能是 2p/N 或者p/N 的整数倍，且截止频率 ω_c 不能任意取值(采样点可能无法触及)

例：利用频率抽样法设计一个频率特性为矩形的理想低通滤波器，截止频率为0.5π，抽样点数为$N=33$，要求滤波器具有线性相位。

解：理想低通频率特性：

$$|H_d(e^{jw})| = \begin{cases} 1 & 0 \le w \le w_c \\ 0 & \text{otherwise} \end{cases}$$

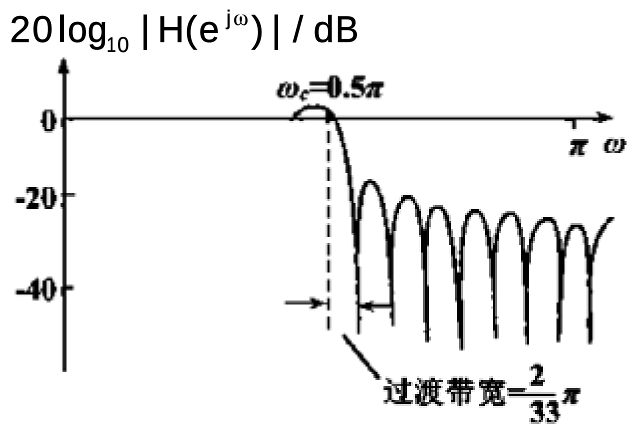


按第一种频率抽样方式，N=33，得抽样点

$$|H(k)| = \begin{cases} 1 & 0 \le k \le \mathrm{Int}\left[\dfrac{Nw_c}{2p}\right] = \dfrac{N-1}{4} = 8 \\ 0 & \mathrm{Int}\left[\dfrac{Nw_c}{2p}\right] + 1 = 9 \le k \le \dfrac{N-1}{2} = 16 \end{cases}$$

得线性相位FIR滤波器的频率响应：

$$H(e^{jw}) = e^{-j16w}\left\{\frac{\sin\left(\dfrac{33w}{2}\right)}{33\sin\left(\dfrac{w}{2}\right)} + \sum_{k=1}^{8}\left[\frac{\sin\left[33\left(\dfrac{w}{2}-\dfrac{kp}{33}\right)\right]}{33\sin\left(\dfrac{w}{2}-\dfrac{kp}{33}\right)} + \frac{\sin\left[33\left(\dfrac{w}{2}+\dfrac{kp}{33}\right)\right]}{33\sin\left(\dfrac{w}{2}+\dfrac{kp}{33}\right)}\right]\right\}$$

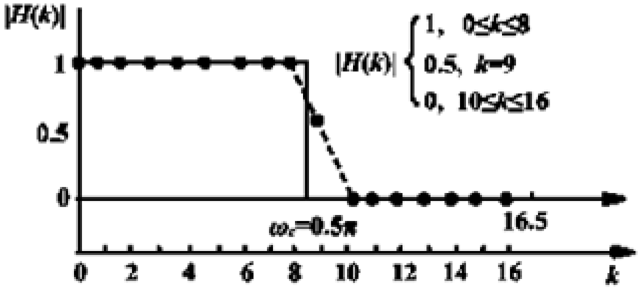


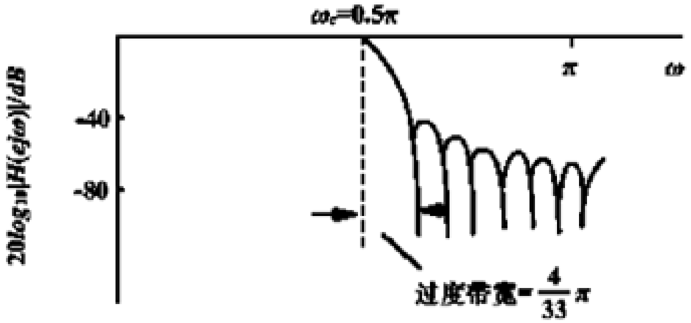
过渡带宽：$2\pi/33$　　阻带衰减：−20dB

AN EXAMPLE

* 增加一点过渡带抽样点

令H(9)=0.5（注意：原本H(9)=0）

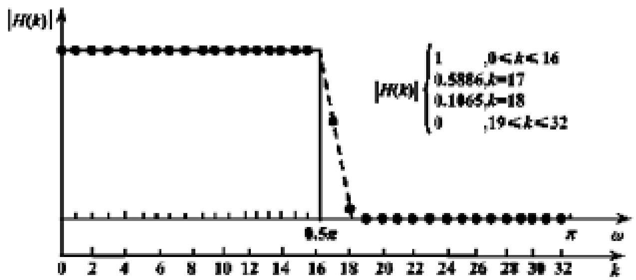


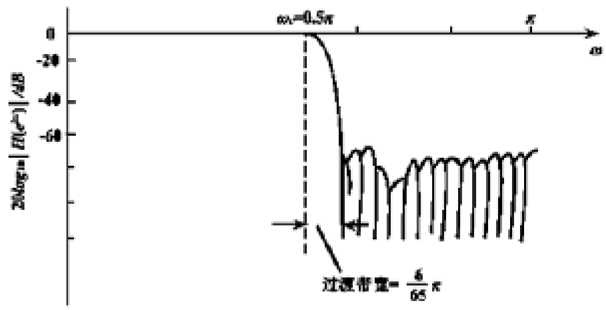


过渡带宽：4π/33　　阻带衰减：-40dB

* 增加两点过渡带抽样点，且增加抽样点数为N=65

令H(17)=0.5886，H(18)=0.1065





过渡带宽：6π/65　　阻带衰减：-60dB

FIR滤波器设计1--往年真题

设理想数字带通滤波器的幅频响应为

$$\left|H_d(e^{jw})\right| = \begin{cases} 1 & p/4 \le |w| \le p/2 \\ 0 & |w| < p/4,\ p/2 \le |w| \le p \end{cases}$$

要求用频率取样法设计相应的$N = 15$时FIR线性相位数字带通滤波器，

(1)确定频率抽样序列$H(k), k = 0, 1, \mathbf{L}, N-1$

(2)确定滤波器的系统函数$H(z)$

(3)给出滤波器的任意一种结构实现形式

02(04-05)

解：

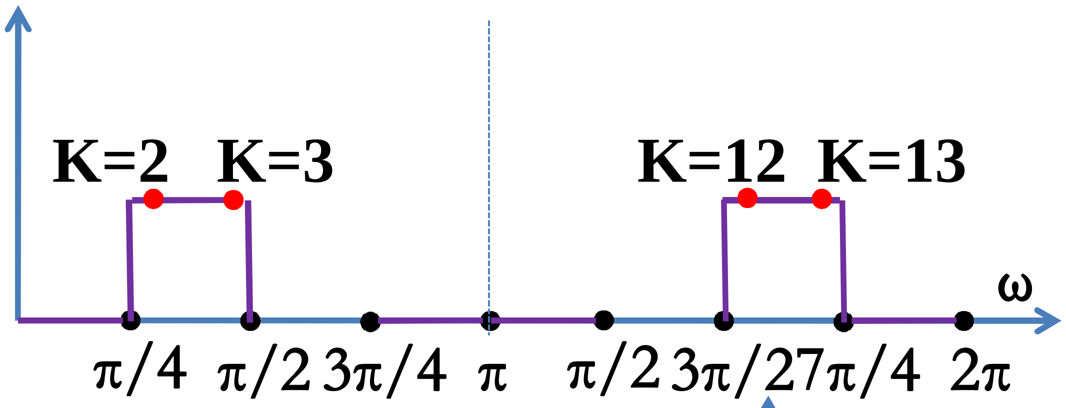
(1) 理想数字带通滤波器的幅频响为

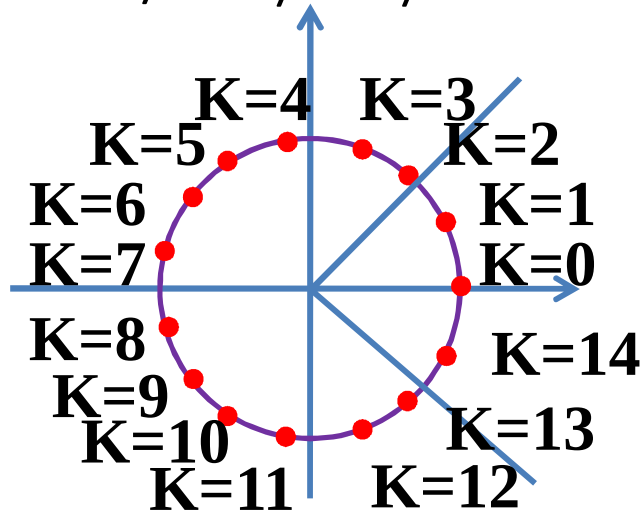
$$H(k)=H_d(e^{jw})\Big|_{w=\frac{2p}{N}k}$$

$$\Delta w=\frac{2p}{N}=\frac{2p}{15}$$

$$\begin{cases} 1<{}^{p/4}\!\big/_{\Delta w}={}^{p/4}\!\big/_{2p/15}=\dfrac{15}{8}<2 \\ 3<{}^{p/2}\!\big/_{\Delta w}={}^{p/2}\!\big/_{2p/15}=\dfrac{15}{4}<4 \\ 11<{}^{3p/2}\!\big/_{\Delta w}={}^{3p/2}\!\big/_{2p/15}=\dfrac{45}{4}<12 \\ 13<{}^{7p/4}\!\big/_{\Delta w}={}^{7p/4}\!\big/_{2p/15}=\dfrac{105}{8}<14 \end{cases}$$

$$\Rightarrow |H_d(k)|=\begin{cases}1, & k=2,3,12,13\\ 0, & \text{其他}\end{cases}$$





$$H(k)=|H_d(k)|e^{-j\frac{N-1}{2}g\frac{2p}{N}k}=e^{-j\frac{14}{15}pk}$$

$$k=2,3,12,13$$

$$H(k)=\begin{cases}e^{-j\frac{14}{15}pk}, & k=2,3,12,13\\ 0, & \text{其他}\end{cases}$$

$$H(k) = \left|H_d(k)\right| e^{-j\frac{N-1}{2}\cdot\frac{2p}{N}k} = e^{-j\frac{14}{15}pk}$$

$$k = 2,3,12,13$$

$$H(k) = \begin{cases} e^{-j\frac{14}{15}pk}, & k = 2,3,12,13 \\ 0, & \text{其他} \end{cases}$$

$$H(0) = H(1) = H(4) = H(5) = H(6) = H(7)$$

$$= H(8) = H(9) = H(10) = H(11) = H(14) = 0$$

$$H(2) = e^{-j\frac{28}{15}p} = e^{j\frac{2}{15}p} = 0.91 + j0.41,$$

$$H(13) = e^{-j\frac{182}{15}p} = e^{-j\frac{2}{15}p} = 0.91 - j0.41$$

$$H(3) = e^{-j\frac{42}{15}p} = e^{-j\frac{12}{15}p} = -0.81 + j0.59,$$

$$H(12) = e^{-j\frac{168}{15}p} = e^{j\frac{12}{15}p} = -0.81 - j0.59$$

$$H(k) = H^*(N-k)$$

或采用P243（5-238）求解

$$H(k)=\begin{cases}\left|H(k)\right|e^{-j\frac{2\pi}{N}k\left(\frac{N-1}{2}\right)} & k=0,\ldots,\frac{N-1}{2}\\ \left|H(k)\right|e^{j\frac{2\pi}{N}(N-k)\left(\frac{N-1}{2}\right)}\ or\ \left|H(N-k)\right|e^{j\frac{2\pi}{N}(N-k)\left(\frac{N-1}{2}\right)} & k=\frac{N+1}{2},\ldots,N-1\end{cases}$$

$$\Rightarrow H(k)=\begin{cases}e^{-j\frac{2\pi}{N}k\left(\frac{N-1}{2}\right)}=e^{-j\frac{14}{15}\pi k},\quad k=2,3\\ e^{j\frac{2\pi}{N}(N-k)\left(\frac{N-1}{2}\right)}=e^{j\frac{14}{15}\pi(15-k)}=e^{-j\frac{14}{15}\pi k},\quad k=12,13\\ 0,\ \text{其他}\end{cases}$$

$$H(0)=H(1)=H(4)=H(5)=H(6)=H(7)$$
$$=H(8)=H(9)=H(10)=H(11)=H(14)=0$$

$$H(2)=e^{-j\frac{28}{15}\pi}=e^{j\frac{2}{15}\pi}=0.91+j0.41,$$

$$H(13)=e^{-j\frac{182}{15}\pi}=e^{-j\frac{2}{15}\pi}=0.91-j0.41$$

$$H(3)=e^{-j\frac{42}{15}\pi}=e^{-j\frac{12}{15}\pi}=-0.81+j0.59,$$

$$H(12)=e^{-j\frac{168}{15}\pi}=e^{j\frac{12}{15}\pi}=-0.81-j0.59$$

$$(2)H(z)=\frac{1-z^{-N}}{N}\sum_{k=0}^{N-1}\frac{H(k)}{1-W_N^{-k}z^{-1}}$$

$$H(z)=\frac{1-z^{-15}}{15}\sum_{k=0}^{14}\frac{e^{-j\frac{14}{15}p\,k}}{1-W_{15}^{-k}z^{-1}}=\frac{1-z^{-15}}{15}\sum_{k=2,3,12,13}\frac{e^{-j\frac{14}{15}p\,k}}{1-e^{-j\frac{2p}{15}k}z^{-1}}$$

$$=\frac{1-z^{-15}}{15}\left(\frac{e^{-j\frac{14}{15}p\,2}}{1-e^{-j\frac{2p}{15}2}z^{-1}}+\frac{e^{-j\frac{14}{15}p\,3}}{1-e^{-j\frac{2p}{15}3}z^{-1}}+\frac{e^{-j\frac{14}{15}p\,12}}{1-e^{-j\frac{2p}{15}12}z^{-1}}+\frac{e^{-j\frac{14}{15}p\,13}}{1-e^{-j\frac{2p}{15}13}z^{-1}}\right)$$

$$=\frac{1-z^{-15}}{15}\left(\left(\frac{e^{-j\frac{14}{15}p\,2}}{1-e^{-j\frac{2p}{15}2}z^{-1}}+\frac{e^{-j\frac{14}{15}p\,13}}{1-e^{-j\frac{2p}{15}13}z^{-1}}\right)+\left(\frac{e^{-j\frac{14}{15}p\,3}}{1-e^{-j\frac{2p}{15}3}z^{-1}}+\frac{e^{-j\frac{14}{15}p\,12}}{1-e^{-j\frac{2p}{15}12}z^{-1}}\right)\right)$$

$$=\frac{1-z^{-15}}{15}\left(\left(\frac{e^{-j\frac{2}{15}p}}{1-e^{-j\frac{4p}{15}}z^{-1}}+\frac{e^{j\frac{2}{15}p}}{1-e^{j\frac{4p}{15}}z^{-1}}\right)+\left(\frac{e^{-j\frac{4}{5}p}}{1-e^{-j\frac{2p}{5}}z^{-1}}+\frac{e^{j\frac{4}{5}p}}{1-e^{j\frac{2p}{5}}z^{-1}}\right)\right)$$

$$
\begin{aligned}
&= \frac{1-z^{-15}}{15}\left(\left(\frac{e^{-j\frac{2}{15}p}}{1-e^{-j\frac{4p}{15}}z^{-1}} + \frac{e^{j\frac{2}{15}p}}{1-e^{j\frac{4p}{15}}z^{-1}}\right) + \left(\frac{e^{-j\frac{4}{5}p}}{1-e^{-j\frac{2p}{5}}z^{-1}} + \frac{e^{j\frac{4}{5}p}}{1-e^{j\frac{2p}{5}}z^{-1}}\right) \right) \\
&= \frac{1-z^{-15}}{15}\left(\frac{e^{-j\frac{2}{15}p}(1-e^{j\frac{4p}{15}}z^{-1}) + e^{j\frac{2}{15}p}(1-e^{-j\frac{4p}{15}}z^{-1})}{(1-e^{-j\frac{4p}{15}}z^{-1})(1-e^{j\frac{4p}{15}}z^{-1})} + \frac{e^{-j\frac{4}{5}p}(1-e^{j\frac{2p}{5}}z^{-1}) + e^{j\frac{4}{5}p}(1-e^{-j\frac{2p}{5}}z^{-1})}{(1-e^{-j\frac{2p}{5}}z^{-1})(1-e^{j\frac{2p}{5}}z^{-1})} \right) \\
&= \frac{1-z^{-15}}{15}\left(\frac{(e^{j\frac{2}{15}p}+e^{-j\frac{2}{15}p}) - (e^{j\frac{2}{15}p}+e^{-j\frac{2}{15}p})z^{-1}}{1-2\cos\frac{4p}{15}z^{-1}+z^{-2}} + \frac{(e^{j\frac{4}{5}p}+e^{-j\frac{4}{5}p}) - (e^{j\frac{2}{5}p}+e^{-j\frac{2}{5}p})z^{-1}}{1-2\cos\frac{2p}{5}z^{-1}+z^{-2}} \right) \\
&= \frac{1-z^{-15}}{15}\left(\frac{2\cos\frac{2p}{15} - 2\cos\frac{2p}{15}z^{-1}}{1-2\cos\frac{4p}{15}z^{-1}+z^{-2}} + \frac{2\cos\frac{4p}{5} - 2\cos\frac{2p}{5}z^{-1}}{1-2\cos\frac{2p}{5}z^{-1}+z^{-2}} \right) \\
&= \frac{1-z^{-15}}{15}\left(\frac{1.83-1.83z^{-1}}{1-1.34z^{-1}+z^{-2}} + \frac{-1.62-0.62z^{-1}}{1-1.83z^{-1}+z^{-2}} \right)
\end{aligned}
$$

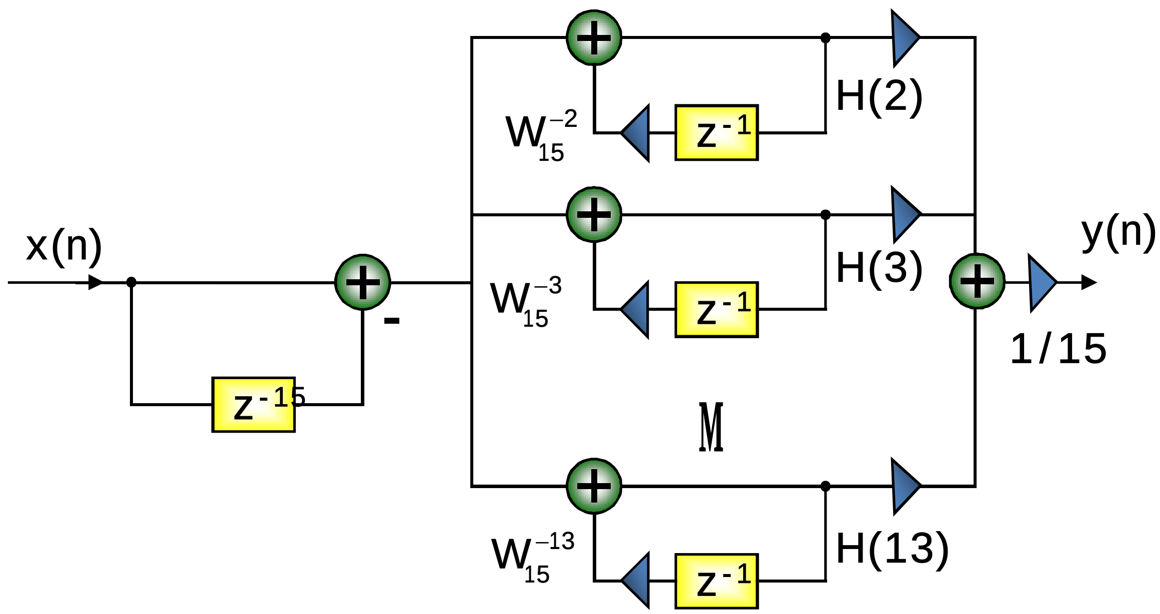
(3)

$$H(2)=e^{-j\frac{28}{15}p}=e^{j\frac{2}{15}p}=0.91+j0.41,$$

$$H(13)=e^{-j\frac{182}{15}p}=e^{-j\frac{2}{15}p}=0.91-j0.41$$

$$H(3)=e^{-j\frac{42}{15}p}=e^{-j\frac{12}{15}p}=-0.81+j0.59,$$

$$H(12)=e^{-j\frac{168}{15}p}=e^{j\frac{12}{15}p}=-0.81-j0.59$$



FIR滤波器设计2--往年真题

设理想数字高通滤波器的幅频响应为

$$\left|H_d(e^{jw})\right| = \begin{cases} 1 & p/2 \le |w| \le p \\ 0 & |w| < p/2 \end{cases}$$

用频率取样法设计相应的$N=11$时FIR线性相位数字高通滤波器，

(1) 确定频域取样序列$H(k), k=0,1,\mathbf{L},N-1$

(2) 确定滤波器的系统函数$H(z)$

(3) 确定滤波器的频率响应$H(e^{jw})$

(4)确定滤波器的单位脉冲响应$h(n)$

(5)给出滤波器的任意一种结构实现形式

注：四舍五入到小数点后2位

03B(05-06)

解：（1）理想数字高通滤波器的幅频响为

$$H(k)=H_d(e^{jw})\Big|_{w=\frac{2p}{N}k}$$

$$\Delta w=\frac{2p}{N}=\frac{2p}{11}$$

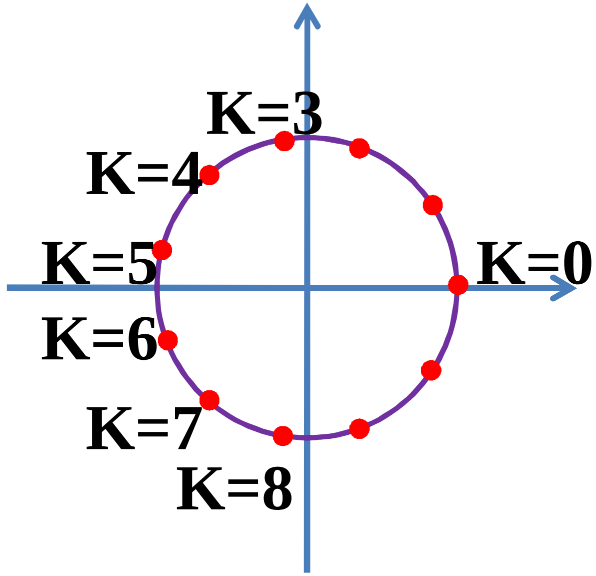
$$\begin{cases} \dfrac{p/2}{\Delta w}=\dfrac{p/2}{2p/11}=\dfrac{11}{4}<3 \\ \dfrac{3p/2}{\Delta w}=\dfrac{3p/2}{2p/11}=\dfrac{33}{4}>8 \end{cases}$$

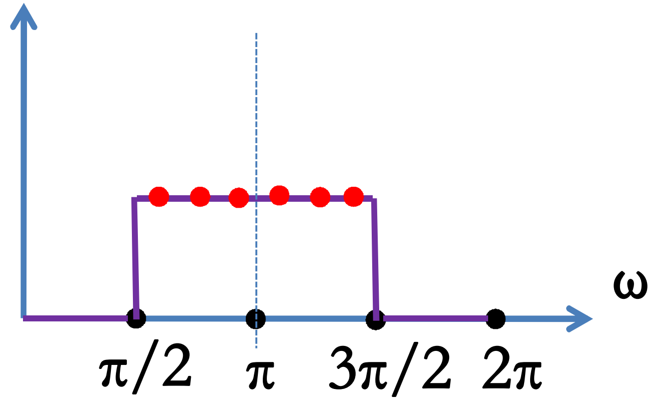
$$\Rightarrow \left|H_d(k)\right|=\begin{cases}1, & 3\le k\le 8\\ 0, & \text{其他}\end{cases}$$

$$H(k)=\left|H_d(k)\right|e^{-j\frac{N-1}{2}\frac{2p}{N}k}=e^{-j\frac{10}{11}pk}$$

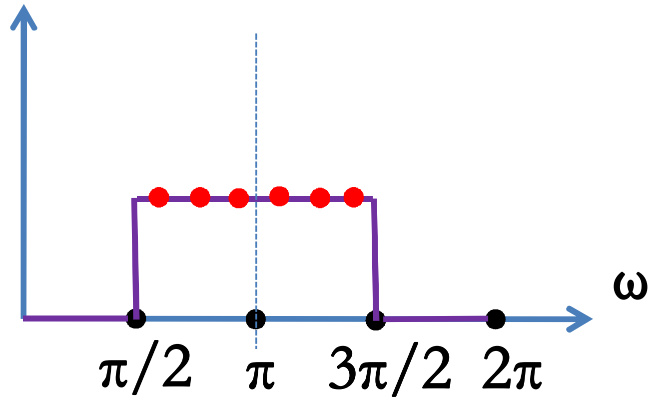
$$k=3,4,5,6,7,8$$

$$H(k)=\begin{cases}e^{-j\frac{10}{11}pk}, & 3\le k\le 8\\ 0, & \text{其他}\end{cases}$$





$$H(k)=\begin{cases} e^{-j\frac{10}{11}p\,k}, & 3\le k\le 8 \\ 0, & \text{其他} \end{cases}$$



$$H(0)=H(1)=H(2)=H(9)=H(10)=0$$

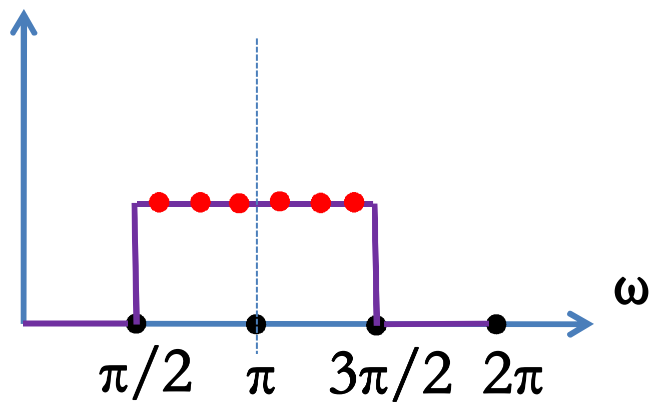
$$H(3)=e^{-j\frac{30}{11}p}=e^{-j\frac{8}{11}p}=-0.65-j0.76,\ \ H(8)=e^{-j\frac{80}{11}p}=e^{j\frac{8}{11}p}=-0.65+j0.76$$

$$H(4)=e^{-j\frac{40}{11}p}=e^{j\frac{4}{11}p}=0.42+j0.91,\quad H(7)=e^{-j\frac{70}{11}p}=e^{-j\frac{4}{11}p}=0.42-j0.91$$

$$H(5)=e^{-j\frac{50}{11}p}=e^{-j\frac{6}{11}p}=-0.14-j0.99,\ \ H(6)=e^{-j\frac{60}{11}p}=e^{j\frac{6}{11}p}=-0.14+j0.99$$

$$H(k)=H^*(N-k)$$

$$(2)H(z)=\frac{1-z^{-N}}{N}\sum_{k=0}^{N-1}\frac{H(k)}{1-W_N^{-k}z^{-1}}$$



$$H(z)=\frac{1-z^{-11}}{11}\sum_{k=0}^{10}\frac{e^{-j\frac{10}{11}pk}}{1-W_{11}^{-k}z^{-1}}=\frac{1-z^{-11}}{11}\sum_{k=0}^{10}\frac{e^{-j\frac{10}{11}pk}}{1-e^{-j\frac{2pk}{11}}z^{-1}}$$

$$=\frac{1-z^{-11}}{11}$$

$$\left(\frac{e^{-j\frac{10}{11}p3}}{1-e^{-j\frac{2p3}{11}}z^{-1}}+\frac{e^{-j\frac{10}{11}p4}}{1-e^{-j\frac{2p4}{11}}z^{-1}}+\frac{e^{-j\frac{10}{11}p5}}{1-e^{-j\frac{2p5}{11}}z^{-1}}+\frac{e^{-j\frac{10}{11}p6}}{1-e^{-j\frac{2p6}{11}}z^{-1}}+\frac{e^{-j\frac{10}{11}p7}}{1-e^{-j\frac{2p7}{11}}z^{-1}}+\frac{e^{-j\frac{10}{11}p8}}{1-e^{-j\frac{2p8}{11}}z^{-1}}\right)$$

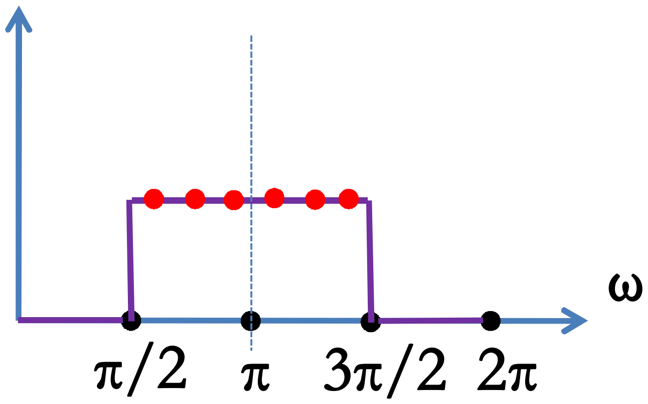
$$=\frac{1-z^{-11}}{11}$$

$$\left(\left(\frac{e^{-j\frac{10}{11}p3}}{1-e^{-j\frac{2p3}{11}}z^{-1}}+\frac{e^{-j\frac{10}{11}p8}}{1-e^{-j\frac{2p8}{11}}z^{-1}}\right)+\left(\frac{e^{-j\frac{10}{11}p4}}{1-e^{-j\frac{2p4}{11}}z^{-1}}+\frac{e^{-j\frac{10}{11}p7}}{1-e^{-j\frac{2p7}{11}}z^{-1}}\right)+\left(\frac{e^{-j\frac{10}{11}p5}}{1-e^{-j\frac{2p5}{11}}z^{-1}}+\frac{e^{-j\frac{10}{11}p6}}{1-e^{-j\frac{2p6}{11}}z^{-1}}\right)\right)$$

$$=\frac{1-z^{-11}}{11}\left(\frac{-1.31+1.31z^{-1}}{1-0.28z^{-1}+z^{-2}}+\frac{0.83-0.83z^{-1}}{1-1.31z^{-1}+z^{-2}}+\frac{-0.28+0.28z^{-1}}{1+1.92z^{-1}+z^{-2}}\right)$$

$$(3)H(e^{jw}) = H(z)\Big|_{z=e^{jw}} = \frac{1-e^{-jw11}}{11}\sum_{k=0}^{10}\frac{H(k)}{1-W_{11}^{-k}e^{-jw}}$$

$$(4)h(n) = \frac{1}{N}\sum_{k=0}^{10}H(k)e^{j\frac{2p}{11}kn}$$



$$h(0) = h(10) = -0.0694$$

$$h(1) = h(9) = 0.0540$$

$$h(2) = h(8) = 0.1094$$

$$h(3) = h(7) = -0.0474$$

$$h(4) = h(6) = -0.3194$$

$$h(5) = 0.5455$$

(5)给出滤波器的任意一种结构实现形式

频率取样型

$$H(3)=e^{-j\frac{30}{11}p}=e^{-j\frac{8}{11}p}=-0.65-j0.76,\quad H(8)=e^{-j\frac{80}{11}p}=e^{j\frac{8}{11}p}=-0.65+j0.76$$

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