

数字信号处理

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第五章 数字滤波器

FIR数字滤波器

窗函数设计法

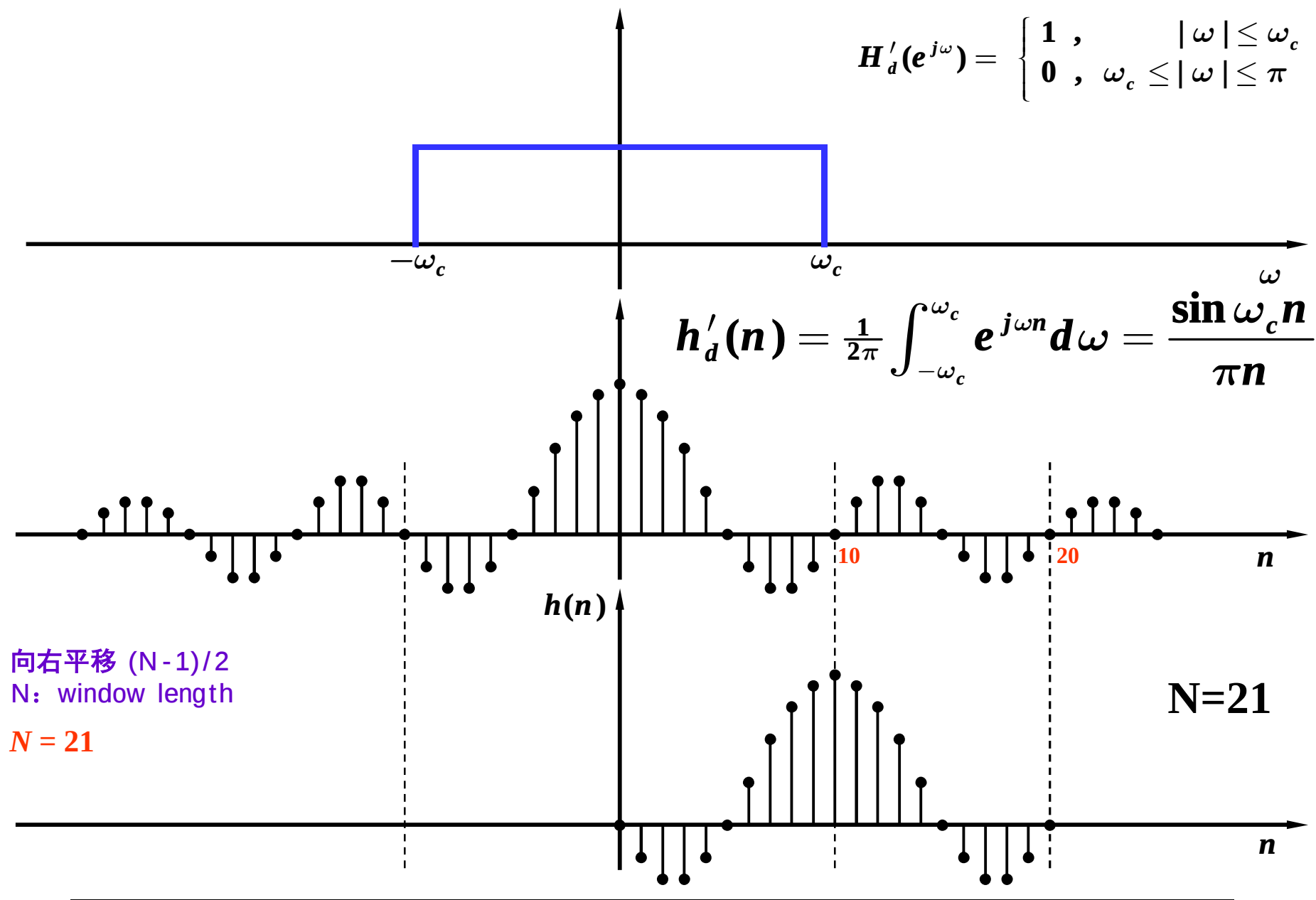
三、窗函数方法：Windowing Method

设计原理：

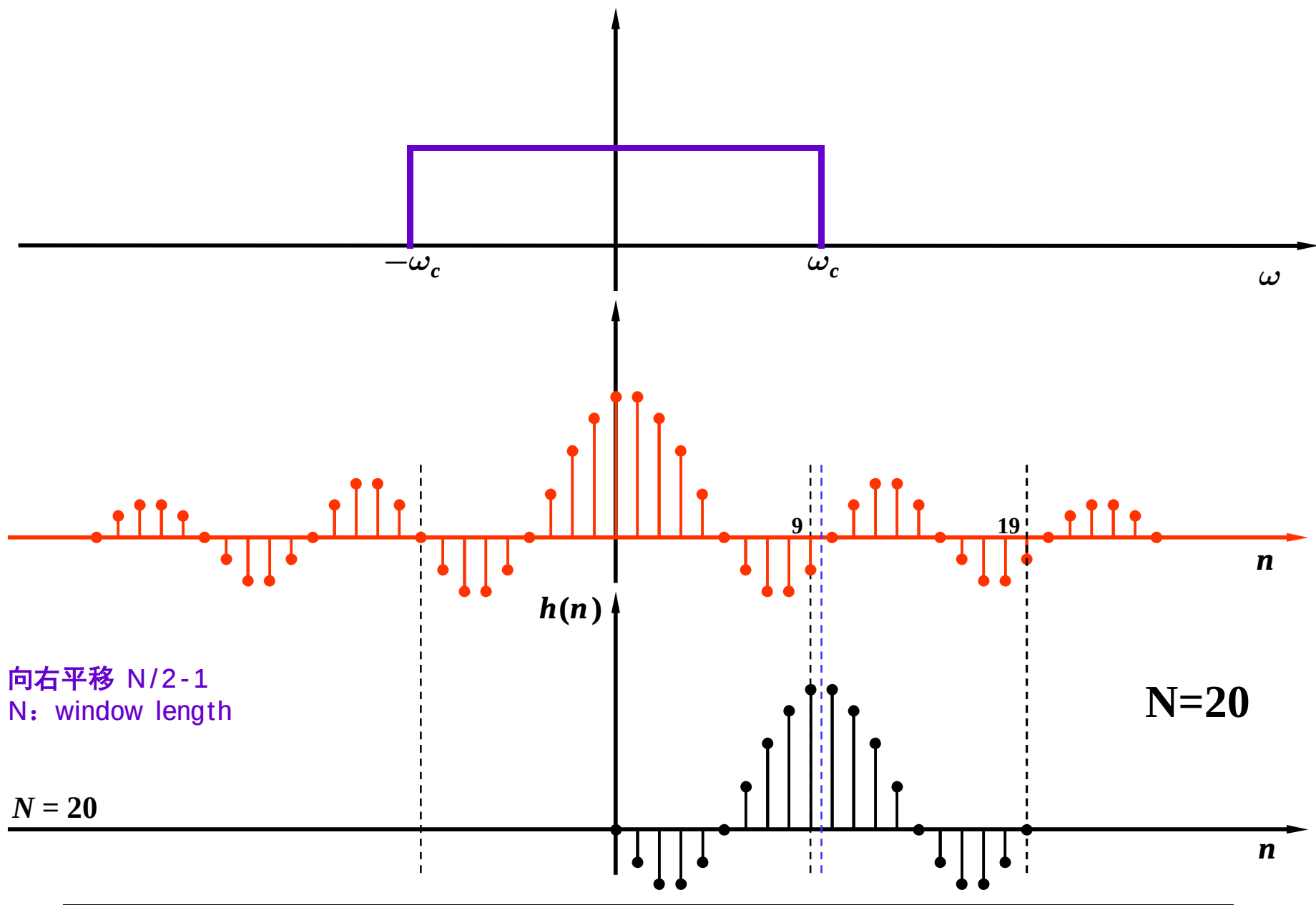
$$H'_d(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \omega_c \\ 0, & \omega_c \leq |\omega| \leq \pi \end{cases}$$
$$h'_d(n) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega n} d\omega = \frac{\sin \omega_c n}{\pi n}$$

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\frac{N-1}{2}\omega}, & |\omega| \leq \omega_c \\ 0, & \omega_c \leq |\omega| \leq \pi \end{cases}$$
$$h_d(n) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{-j\frac{N-1}{2}\omega} e^{j\omega n} d\omega = \frac{\sin \left[\omega_c \left(n - \frac{N-1}{2} \right) \right]}{\pi \left(n - \frac{N-1}{2} \right)}$$

$$h(n) = h_d(n)R_N(n) = \begin{cases} h_d(n), & 0 \leq n \leq N-1 \\ 0, & \text{其他} \end{cases}$$



EXAMPLE: 截止频率为 ω_c 的线性相位理想低通滤波器: $h_d(n)$ 偶对称, 奇数点



EXAMPLE: 截止频率为 ω_c 的线性相位理想低通滤波器: $h_d(n)$ 偶对称, 偶数点

矩形窗截断的影响：

$$h(n) = h_d(n)R_N(n)$$

$$W_R(e^{j\omega}) \Leftrightarrow R_N(n)$$

$$H(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) W_R[e^{j(\omega-\theta)}] d\theta$$

$$\left\{ \begin{array}{l} W_R(e^{j\omega}) = \frac{\sin\left(\frac{\omega N}{2}\right)}{\sin\frac{\omega}{2}} e^{-j\omega\left(\frac{N-1}{2}\right)} = W_R(\omega) e^{-j\omega\alpha} \\ \alpha = \frac{N-1}{2}, W_R(\omega) = \frac{\sin\left(\frac{\omega N}{2}\right)}{\sin\frac{\omega}{2}} \end{array} \right.$$

矩形窗截断的影响：

$$\left\{ \begin{array}{l} H_d(e^{j\omega}) = H_d(\omega)e^{-j\omega\alpha} \\ H_d(\omega) = \begin{cases} 1, & |\omega| \leq \omega_c \\ 0, & \omega_c \leq |\omega| \leq \pi \end{cases} \end{array} \right.$$

$$\begin{aligned} H(e^{j\omega}) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\theta)e^{-j\theta\alpha}W_R(\omega - \theta)e^{j(\omega - \theta)}d\theta \\ &= e^{-j\omega\alpha} \left[\frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\theta)W_R(\omega - \theta)d\theta \right] \\ &= H(\omega)e^{-j\omega\alpha} \end{aligned}$$

$$H(\omega) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\theta)W_R(\omega - \theta)d\theta$$

截断效应：吉布斯现象 用加窗技术减小截断效应

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega}, & |\omega| \leq \omega_c \\ 0, & \omega_c \leq |\omega| \leq \pi \end{cases} = H_d(\omega)e^{-j\alpha\omega}, \alpha = \frac{N-1}{2}$$

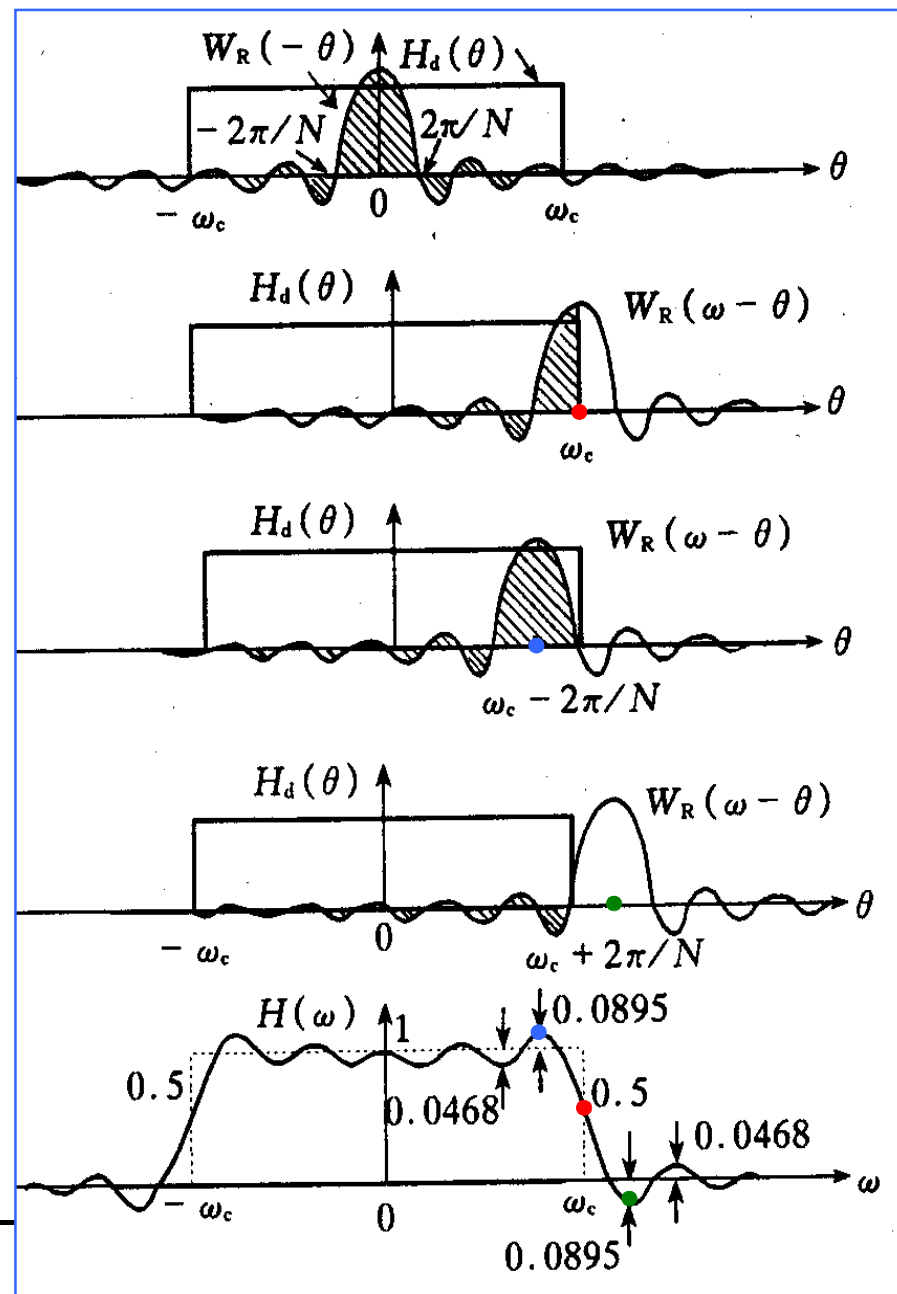
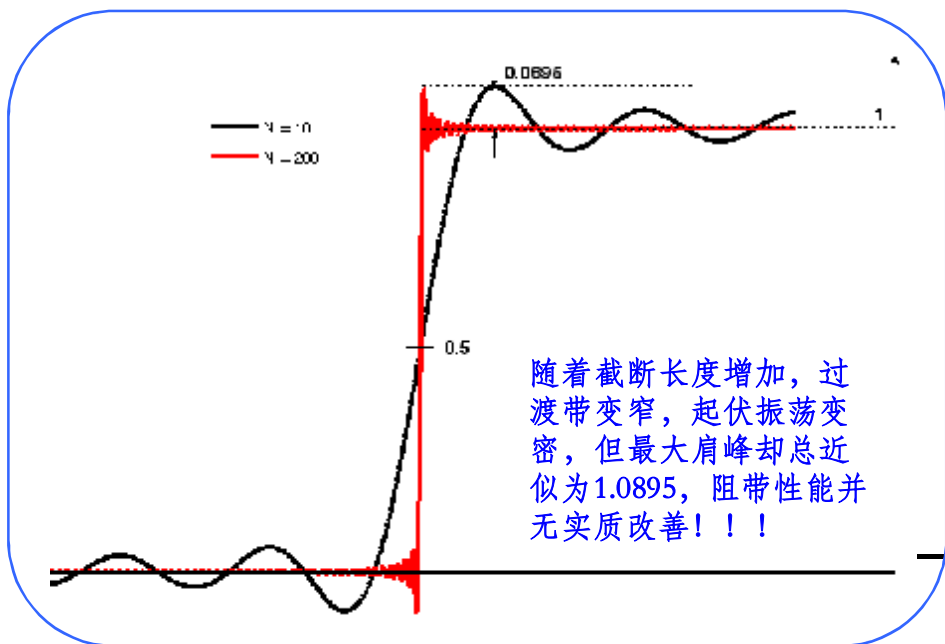
$$H_d(\omega) = \begin{cases} 1, & |\omega| \leq \omega_c \\ 0, & \omega_c \leq |\omega| \leq \pi \end{cases}$$

N: 窗长

$$W_R(e^{j\omega}) = \frac{\sin(\omega N/2)}{\sin(\omega/2)} e^{-j\alpha\omega} = W_R(\omega) e^{-j\alpha\omega}$$

$$H(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\theta}) W_R[e^{j(\omega-\theta)}] d\theta$$

$$= \left[\frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(0) W_R(\omega-0) d\theta \right] e^{-j\alpha\omega} = H(\omega) e^{-j\alpha\omega}$$

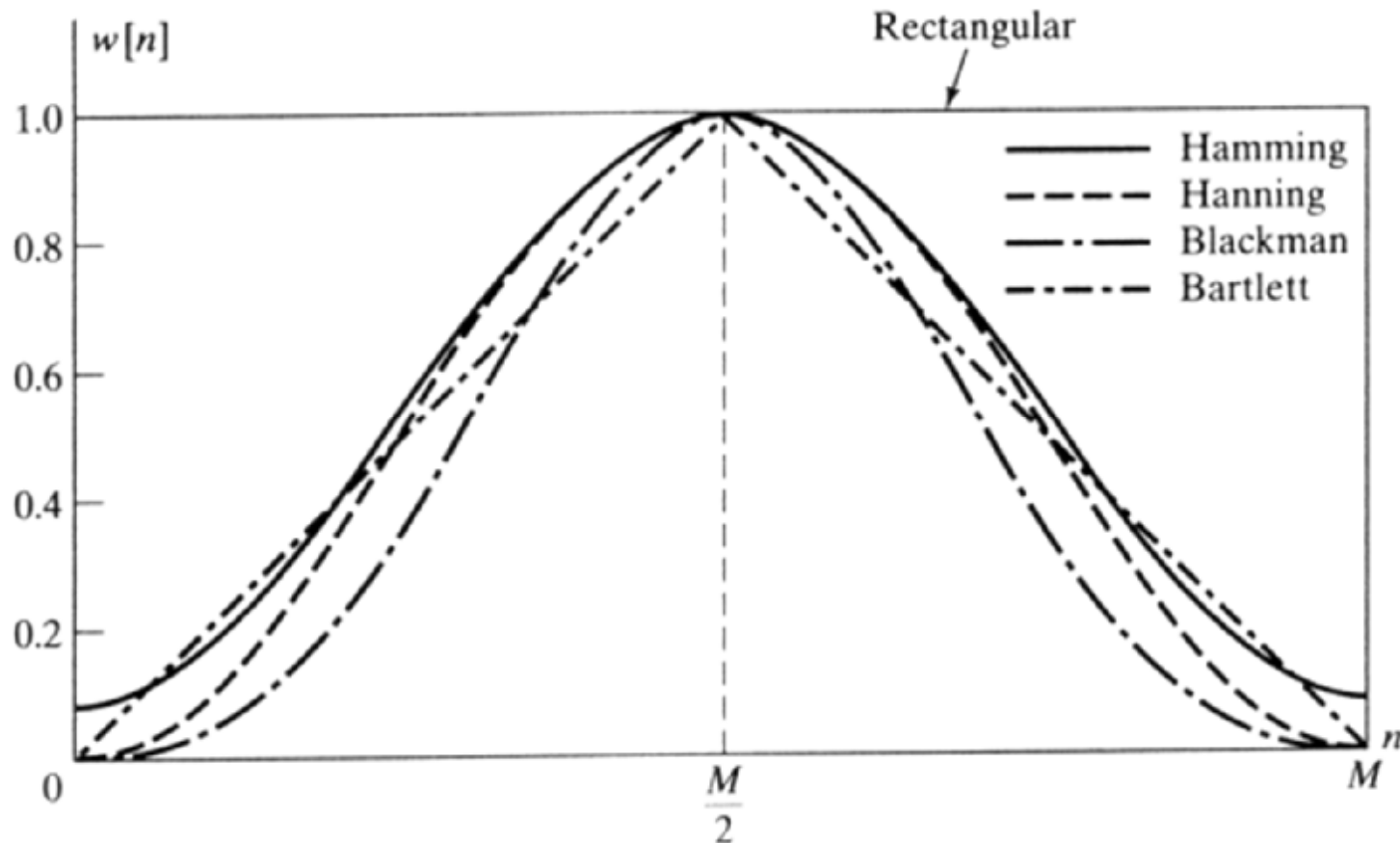


Window Functions for FIR Filter Design

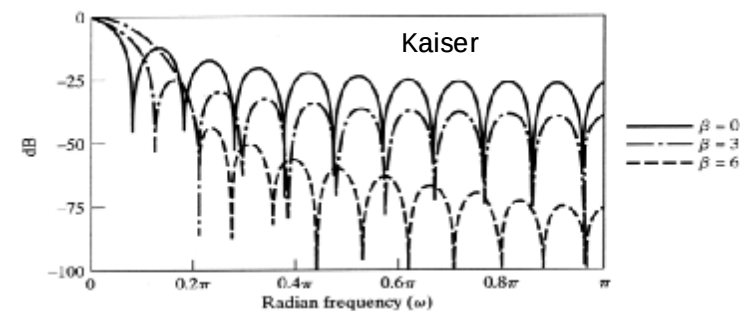
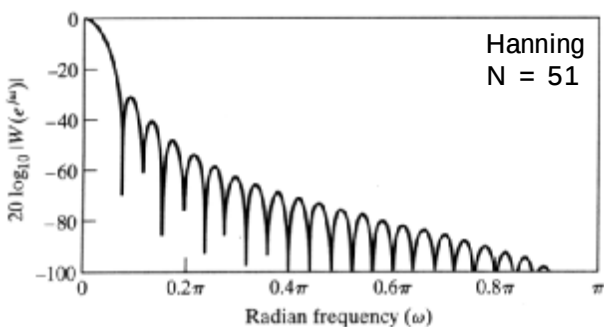
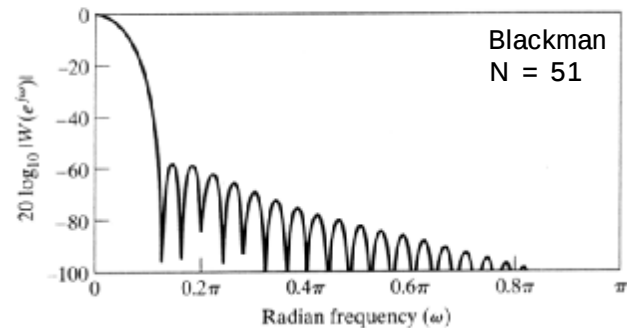
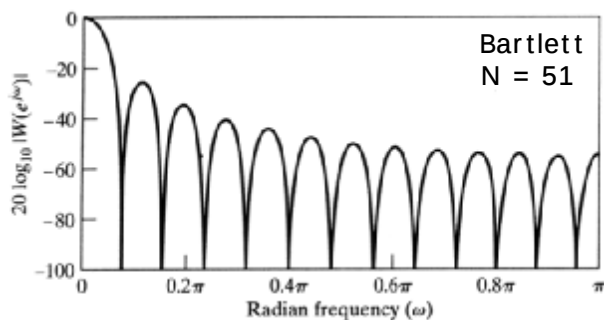
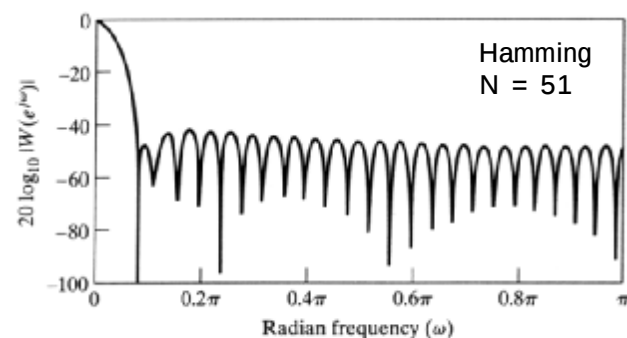
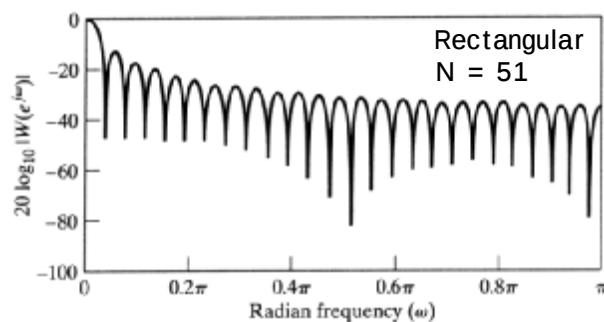
Window Type	Time-Domain Sequence
Rectangular	$w[n] = \begin{cases} 1, & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$
Bartlett (Triangular)	$w[n] = \begin{cases} 2n/M, & 0 \leq n \leq M/2 \\ 2-2n/M, & M/2 < n \leq M \\ 0, & \text{otherwise} \end{cases}$
Hanning	$w[n] = \begin{cases} 0.5 - 0.5\cos(2\pi n/M), & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$
Hamming	$w[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M), & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$
Blackman	$w[n] = \begin{cases} 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M), & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$
Kaiser	$w[n] = \begin{cases} I_0[\beta(1 - \{(n - \alpha)/\alpha\}^2)^{1/2}] / I_0(\beta), & 0 \leq n \leq M, \alpha = M/2 \\ 0, & \text{otherwise} \end{cases}$

$I_0(\cdot)$ is zero order modified Bessel function of the first kind, β is window shape parameter. $M = N-1$.

Shape of commonly used window functions.



主瓣宽度 v. s. 副瓣高度



三、窗函数方法：Windowing Method

设计步骤：

(1) 给定要求的频率响应函数 $H_d(e^{j\omega})$

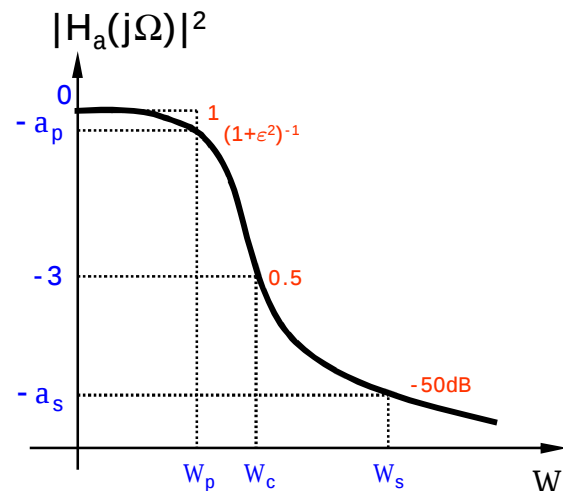
(2) 根据 $h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$ 计算 $h_d(n)$

(3) 根据过渡带及阻带最小衰减要求，选定窗和 N

(4) 根据 $h(n) = h_d(n)R_N(n)$ 求得 $h(n)$

线性相位FIR低通滤波器的设计

例：设计一个线性相位FIR低通滤波器，满足下列条件：抽样频率为15KHz；通带截止频率为1.5KHz；阻带起始频率为3KHz；阻带衰减不小于50dB，幅度特性如右图所示



解：1) 确定模拟指标对应的数字频率

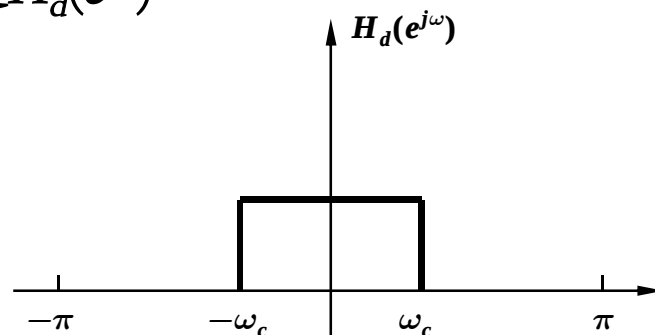
$$\omega_p = 2\pi f_p / F_s = 0.2\pi; \omega_s = 2\pi f_s / F_s = 0.4\pi$$

$$\alpha_s = -50\text{dB}$$

2) 根据过渡带设定选截止频率 ω_c ，由此确定 $H_d(e^{j\omega})$

$$\omega_c = 2\pi \left[\frac{1}{2}(f_p + f_s) \right] / F_s = 0.3\pi; \tau = \frac{N-1}{2}$$

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\tau} & -\omega_c \leq \omega \leq \omega_c \\ 0 & -\pi \leq \omega \leq -\omega_c, \omega_c \leq \omega \leq \pi \end{cases}$$

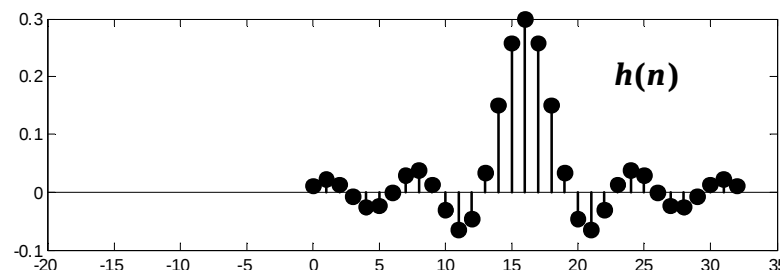
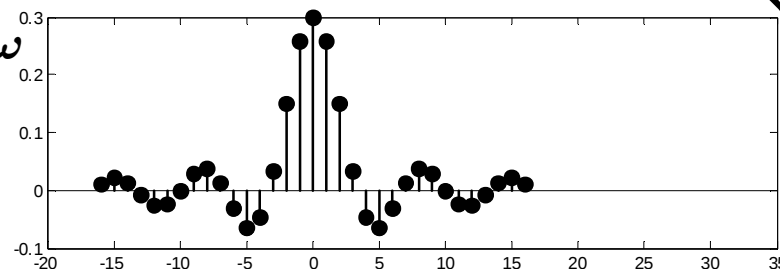


线性相位FIR低通滤波器的设计

$$h_d(n) = \text{IDTFT} \{H_d(e^{j\omega})\} = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-j\omega\tau} e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega(n-\tau)} d\omega$$

$$= \begin{cases} \frac{1}{\pi(n-\tau)} \sin[\omega_c(n-\tau)] & n \neq \tau \\ \frac{\omega_c}{\pi} & n = \tau \end{cases}$$



3) 求 $h_d(n)$

4) 据阻带衰减要求选择窗函数: 由 $\alpha_s = 50\text{dB}$, 确定海明窗 (-53dB)

$$w(n) = \left[0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right) \right] R_N(n)$$

5) 据过渡带宽要求确定窗长 N (海明窗: $\Delta\omega = 6.6\pi/N$)

$$\Delta\omega = 2\pi(f_s - f_p) / F_s = 0.2\pi; N = A / \Delta\omega = 6.6\pi / 0.2\pi = 33$$

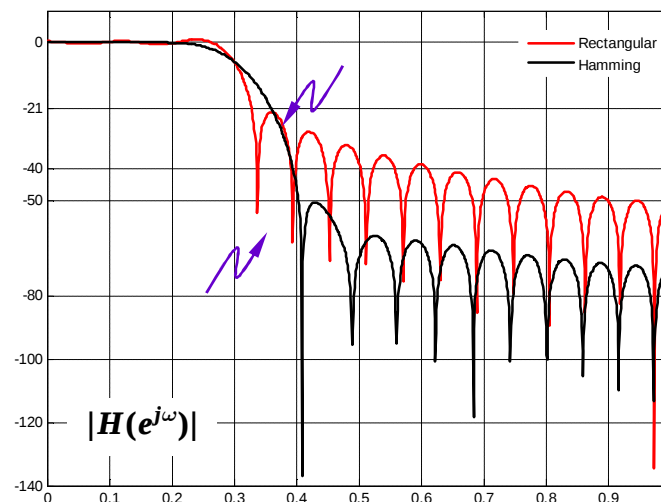
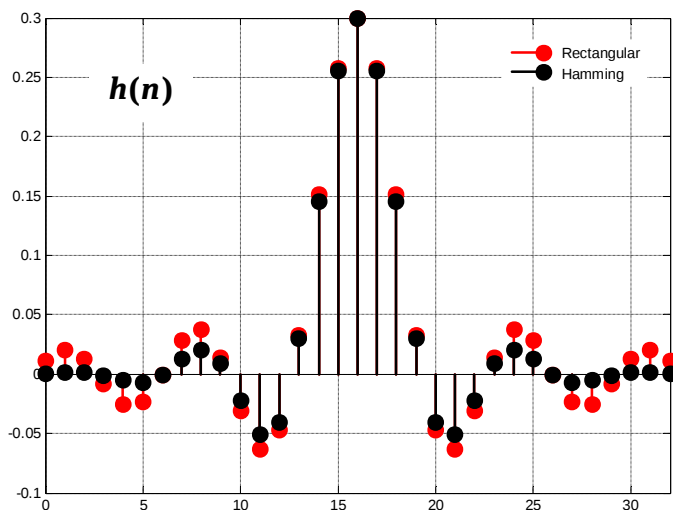
线性相位FIR低通滤波器的设计

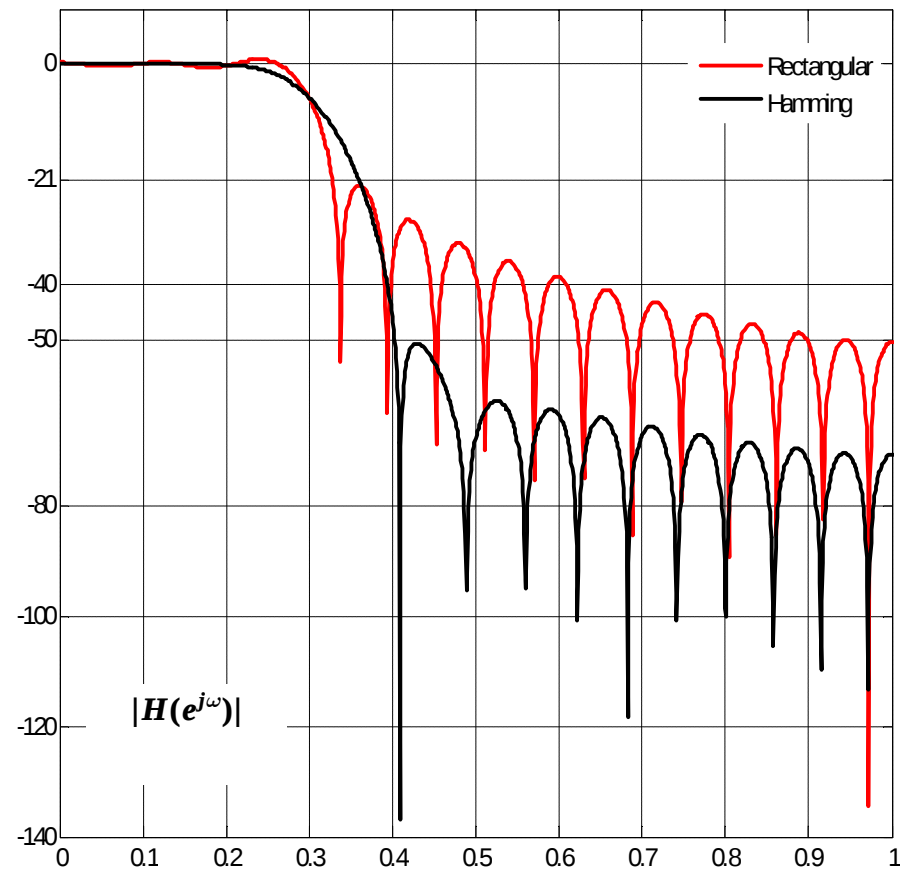
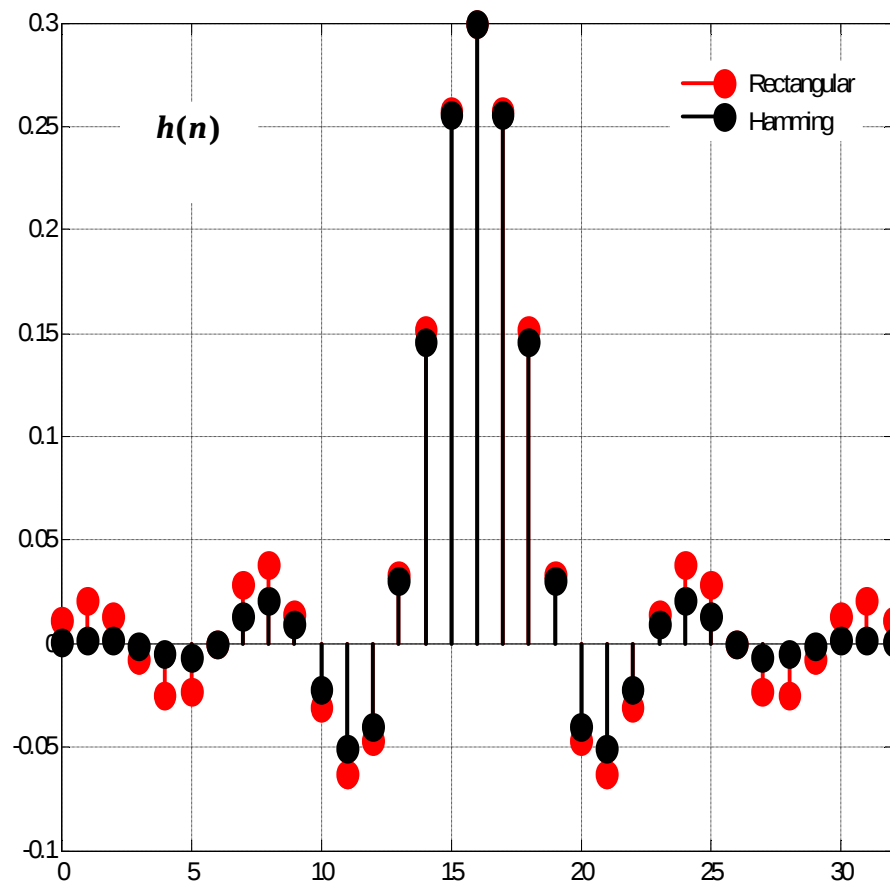
6) 确定FIR滤波器的单位抽样响应 $h(n)$

$$\tau = (N - 1) / 2 = 16$$

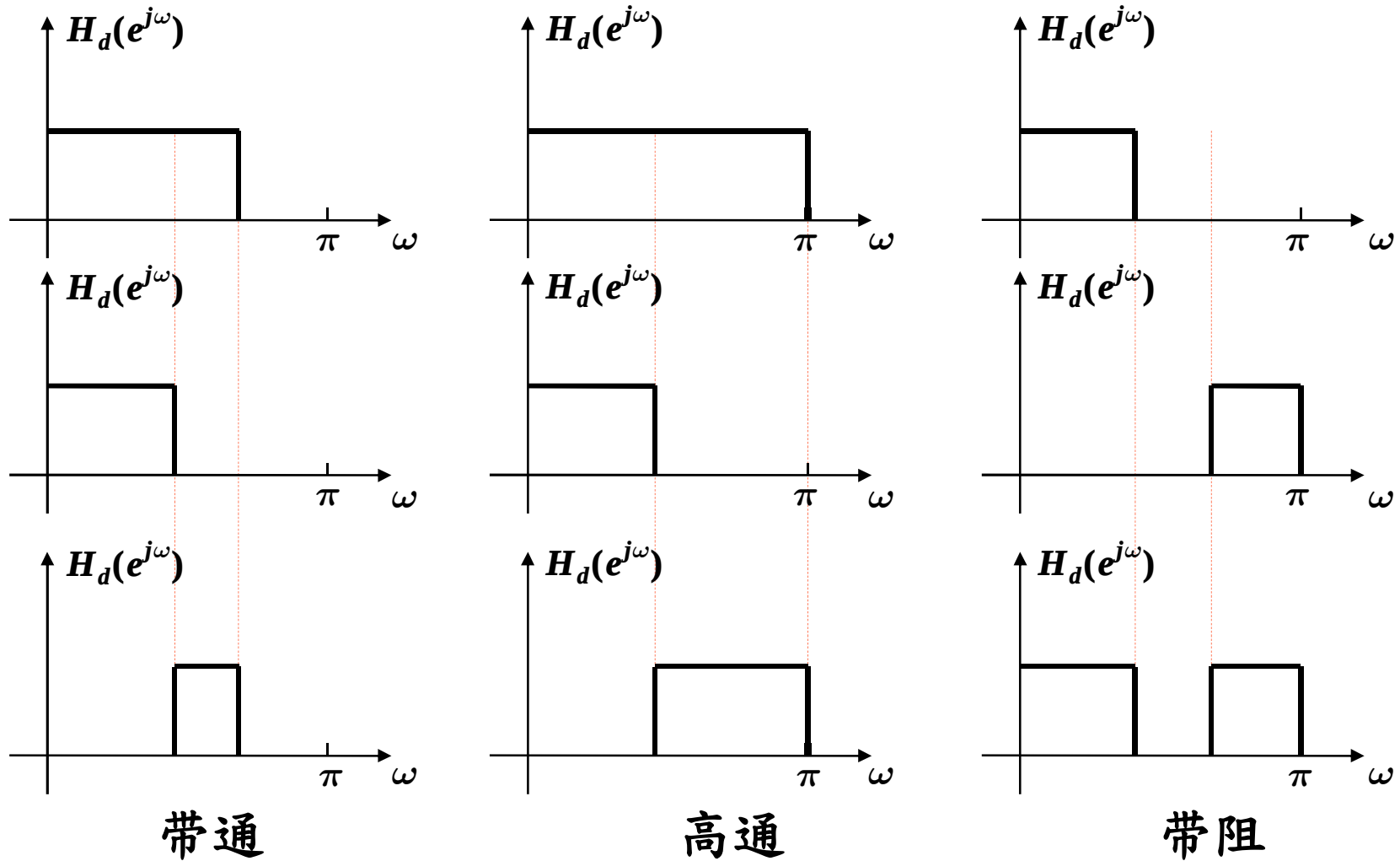
$$\begin{aligned} h(n) &= h_d(n)w(n) \\ &= \frac{\sin\left[0.3\pi(n - 16)\right]}{\pi(n - 16)} \cdot \left[0.54 - 0.46\cos\frac{\pi n}{16}\right] R_{33}(n) \end{aligned}$$

7) 求 $H(e^{j\omega})$ ，并检验性能是否满足预定指标。若不满足，则改变 N 或窗形状重新设计





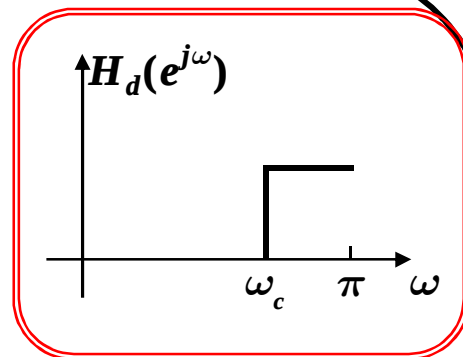
From low pass to band pass, high pass, band stop ...



线性相位FIR高通滤波器的设计公式

— 理想高通的频响：

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\tau} & \omega_c \leq |\omega| \leq \pi \\ 0 & \text{otherwise} \end{cases} \quad \tau = \frac{N-1}{2}$$



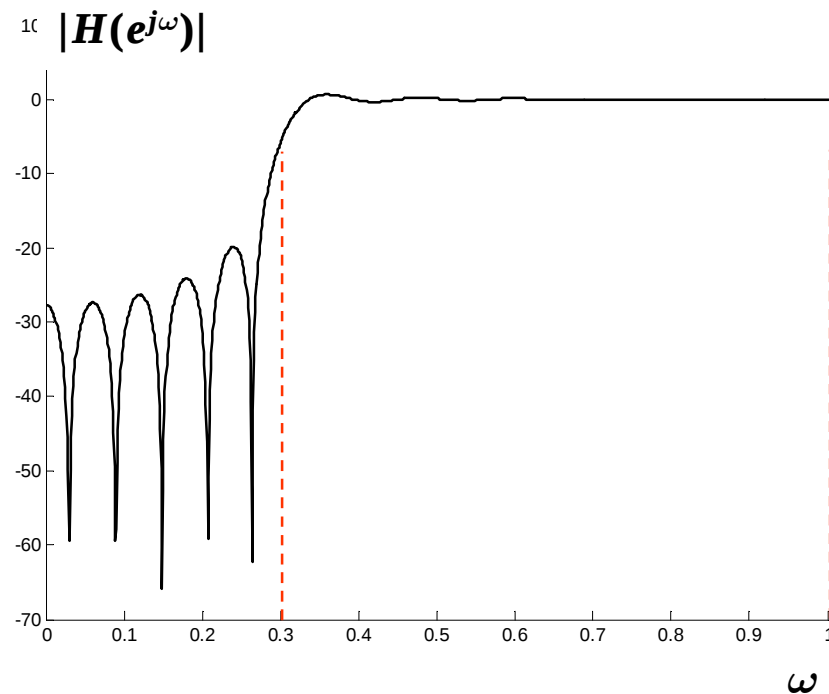
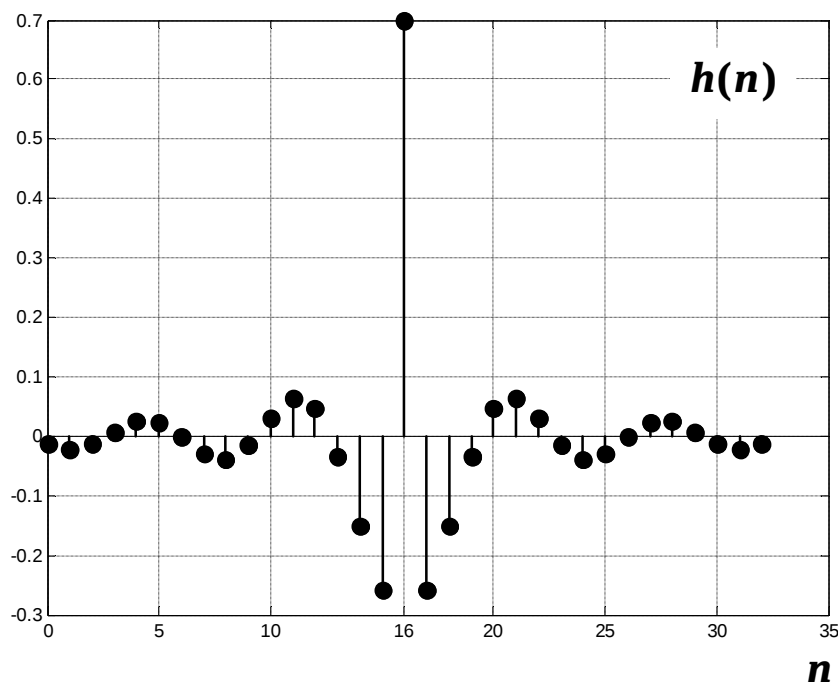
其单位抽样响应：

$$\begin{aligned} h_d(n) &= \frac{1}{2\pi} \left[\int_{-\pi}^{-\omega_c} e^{j\omega(n-\tau)} d\omega + \int_{\omega_c}^{\pi} e^{j\omega(n-\tau)} d\omega \right] \\ &= \begin{cases} \frac{1}{\pi(n-\tau)} \left\{ \sin[\pi(n-\tau)] - \sin[\omega_c(n-\tau)] \right\} & n \neq \tau \\ \frac{1}{\pi} (\pi - \omega_c) & n = \tau \end{cases} \end{aligned}$$

高通滤波器(ω_c) = 全通滤波器 - 低通滤波器(ω_c)

线性相位FIR高通滤波器的设计公式

$$\frac{\sin[\pi(n-16)] - \sin[0.3\pi(n-16)]}{\pi(n-16)}$$

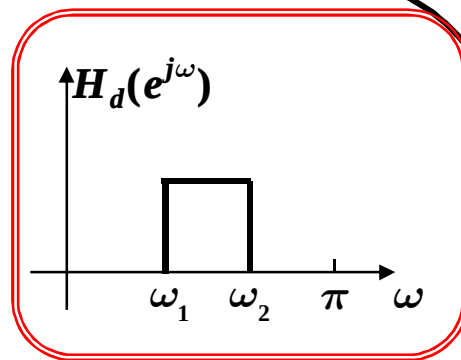


N=33
 0.3π

线性相位FIR带通滤波器的设计公式

— 理想带通的频响：

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\tau} & 0 < \omega_1 \leq |\omega| \leq \omega_2 < \pi \\ 0 & \text{otherwise} \end{cases} \quad \tau = \frac{N-1}{2}$$



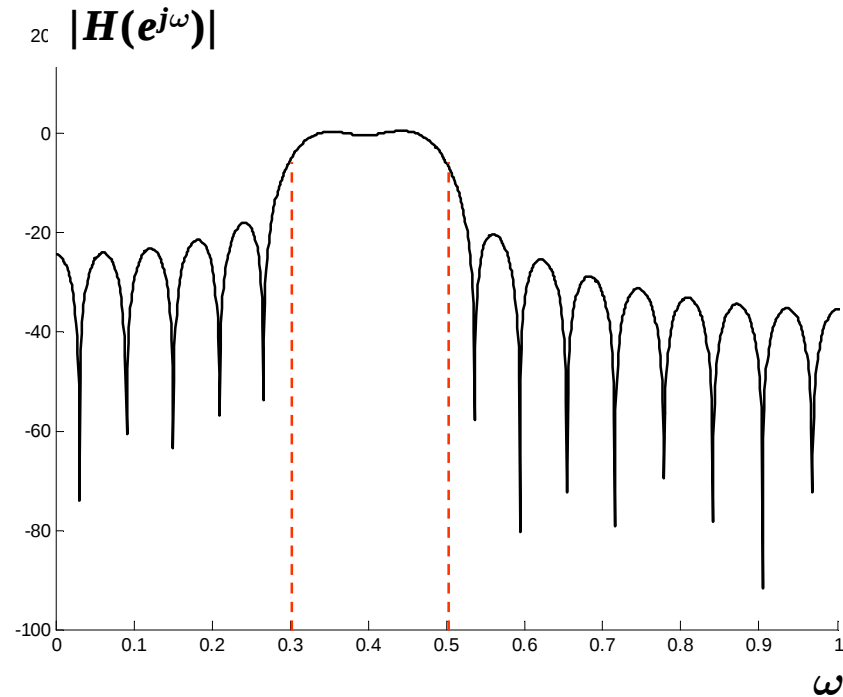
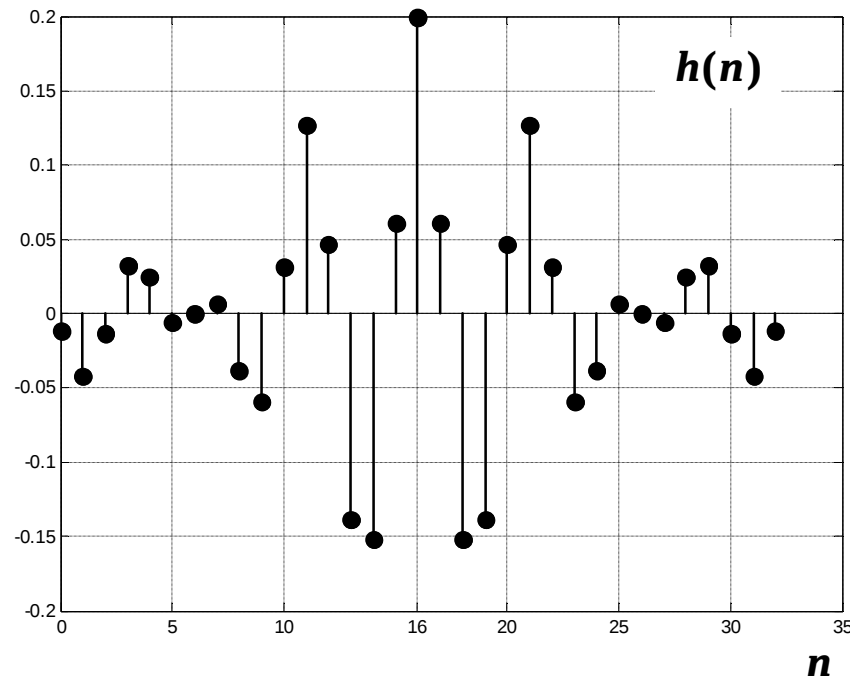
其单位抽样响应：

$$\begin{aligned} h_d(n) &= \frac{1}{2\pi} \left[\int_{-\omega_2}^{-\omega_1} e^{j\omega(n-\tau)} d\omega + \int_{\omega_1}^{\omega_2} e^{j\omega(n-\tau)} d\omega \right] \\ &= \begin{cases} \frac{1}{\pi(n-\tau)} \left\{ \sin[\omega_2(n-\tau)] - \sin[\omega_1(n-\tau)] \right\} & n \neq \tau \\ \frac{1}{\pi} (\omega_2 - \omega_1) & n = \tau \end{cases} \end{aligned}$$

带通滤波器(ω_1, ω_2) = 低通滤波器(ω_2) - 低通滤波器(ω_1)

线性相位FIR带通滤波器的设计公式

$$\frac{\sin[0.5\pi(n-16)] - \sin[0.3\pi(n-16)]}{\pi(n-16)}$$

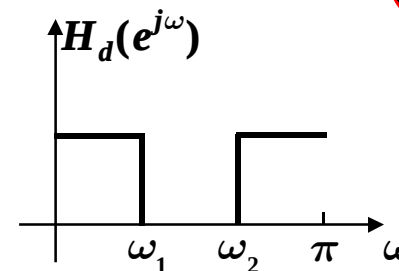


N=33
 $0.3\pi, 0.5\pi$

线性相位FIR带阻滤波器的设计公式

— 理想带阻的频响:

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\tau} & 0 \leq |\omega| \leq \omega_1, \omega_2 \leq |\omega| \leq \pi \\ 0 & \text{otherwise} \end{cases} \quad \tau = \frac{N-1}{2}$$



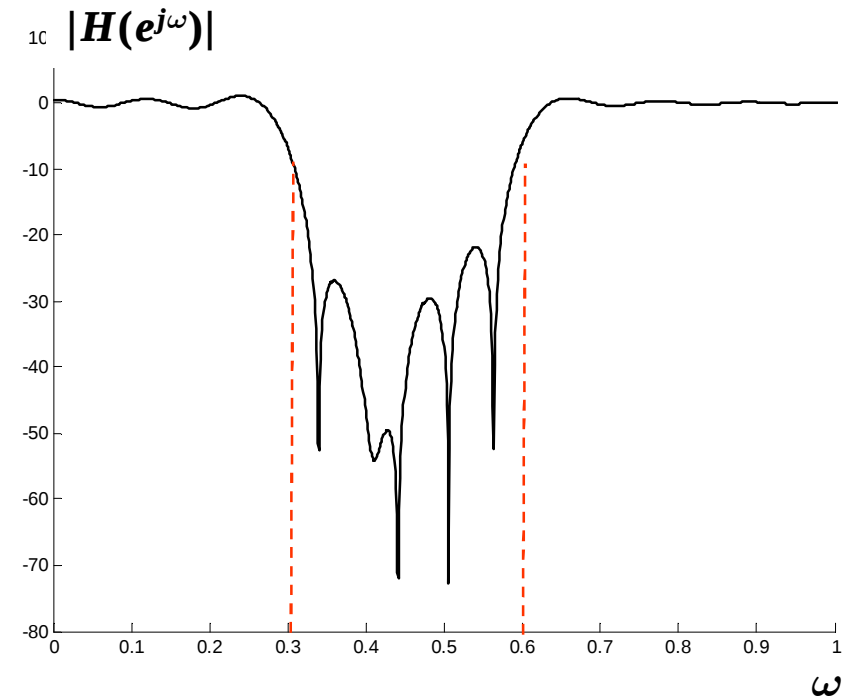
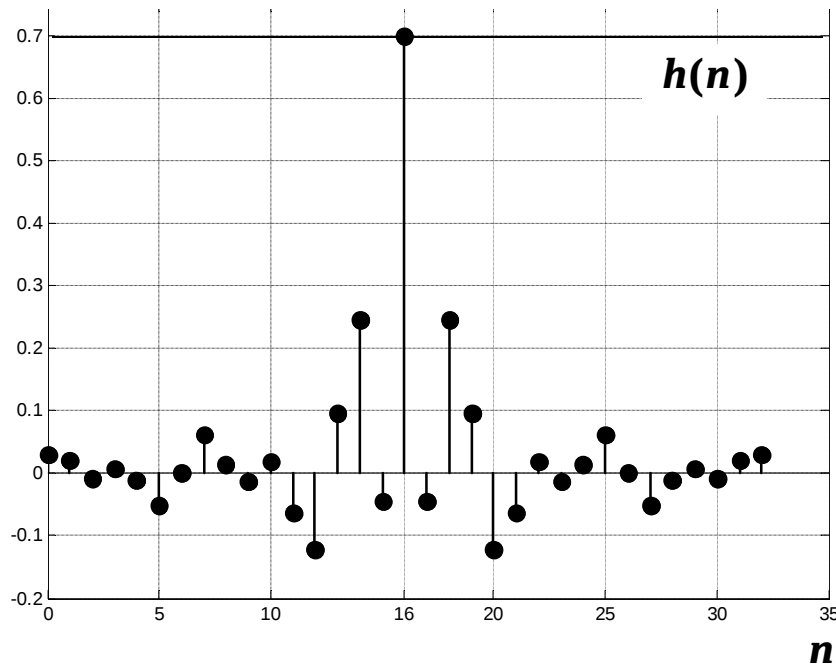
其单位抽样响应:

$$\begin{aligned} h_d(n) &= \frac{1}{2\pi} \left[\int_{-\pi}^{-\omega_2} e^{j\omega(n-\tau)} d\omega + \int_{-\omega_1}^{\omega_1} e^{j\omega(n-\tau)} d\omega + \int_{\omega_2}^{\pi} e^{j\omega(n-\tau)} d\omega \right] \\ &= \begin{cases} \frac{1}{\pi(n-\tau)} \left\{ \sin[\pi(n-\tau)] + \sin[\omega_1(n-\tau)] - \sin[\omega_2(n-\tau)] \right\} & n \neq \tau \\ \frac{1}{\pi} (\pi + \omega_1 - \omega_2) & n = \tau \end{cases} \end{aligned}$$

带阻滤波器(ω_1, ω_2) = 高通滤波器(ω_2) + 低通滤波器(ω_1)

线性相位FIR带阻滤波器的设计公式

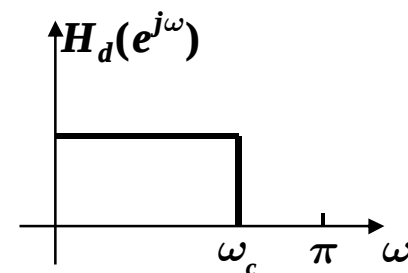
$$\frac{\sin[\pi(n-16)] + \sin[0.3\pi(n-16)] - \sin[0.6\pi(n-16)]}{\pi(n-16)}$$



N=33
 $0.3\pi, 0.6\pi$

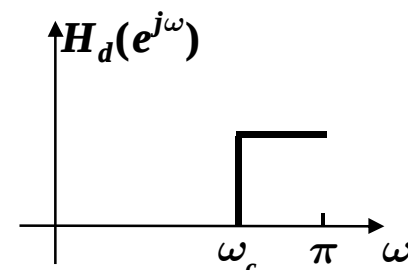
低通滤波器(ω_c)

$$h_d(n) = \begin{cases} \frac{1}{\pi(n-\tau)} \sin[\omega_c(n-\tau)] & n \neq \tau \\ \frac{\omega_c}{\pi} & n = \tau \end{cases}$$



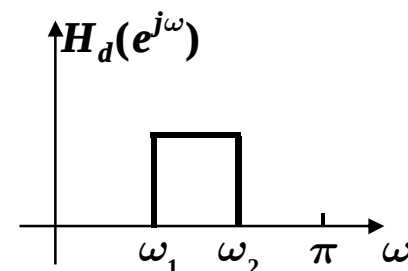
高通滤波器(ω_c) = 全通滤波器 - 低通滤波器(ω_c)

$$h_d(n) = \begin{cases} \frac{1}{\pi(n-\tau)} \left\{ \sin[\pi(n-\tau)] - \sin[\omega_c(n-\tau)] \right\} & n \neq \tau \\ \frac{1}{\pi}(\pi - \omega_c) & n = \tau \end{cases}$$



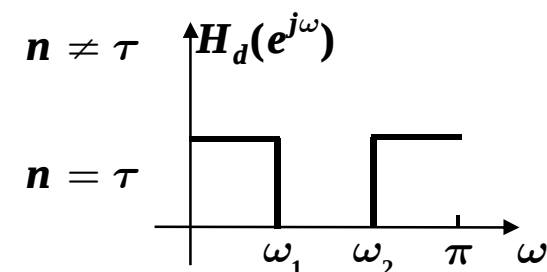
带通滤波器(ω_1, ω_2) = 低通滤波器(ω_2) - 低通滤波器(ω_1)

$$h_d(n) = \begin{cases} \frac{1}{\pi(n-\tau)} \left\{ \sin[\omega_2(n-\tau)] - \sin[\omega_1(n-\tau)] \right\} & n \neq \tau \\ \frac{1}{\pi}(\omega_2 - \omega_1) & n = \tau \end{cases}$$



带阻滤波器(ω_1, ω_2) = 高通滤波器(ω_2) + 低通滤波器(ω_1)

$$h_d(n) = \begin{cases} \frac{1}{\pi(n-\tau)} \left\{ \sin[\pi(n-\tau)] + \sin[\omega_1(n-\tau)] - \sin[\omega_2(n-\tau)] \right\} & n \neq \tau \\ \frac{1}{\pi}(\pi + \omega_1 - \omega_2) & n = \tau \end{cases}$$





FIR滤波器设计1—习题集P108

用矩形窗函数方法设计一个FIR线性相位数字低通滤波器，
已知 $w_c = 0.5p$, $N = 21$ 。

- (1) 确定单位抽样响应序列 $h(n)$, $n = 0, 1, \dots, N-1$
- (2) 确定滤波器的系统函数 $H(z)$
- (3) 确定滤波器的频率响应 $H(e^{jw})$
- (4) 给出滤波器的任意一种结构实现形式

理想数字低通滤波器的幅频响应为

$$|H_d(e^{jw})| = \begin{cases} 1 & -w_c \leq |w| \leq w_c \\ 0 & w_c < |w| < p \end{cases}$$



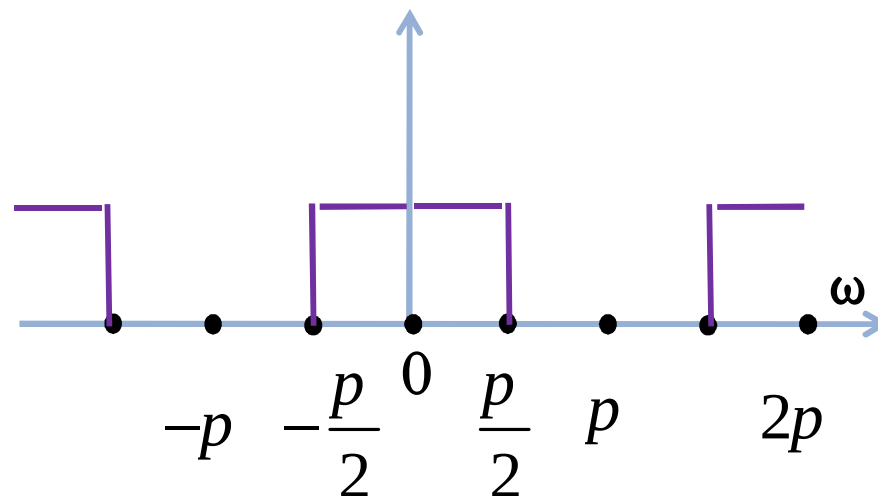
解：理想数字低通滤波器的幅频响应为

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega a} & -\omega_c \leq \omega \leq \omega_c \\ 0 & \text{其他} \end{cases}$$

$$\Rightarrow a = \frac{N-1}{2} = 10, \quad \omega_c = \frac{p}{2}$$

$$(1) h_d(n) = \begin{cases} \frac{1}{p(n-t)} \sin[\omega_c(n-t)] & n \neq t \\ \frac{\omega_c}{p} & n = t \end{cases} = \begin{cases} \frac{1}{p(n-10)} \sin[\frac{p}{2}(n-t)] & n \neq 10 \\ \frac{1}{2} & n = 10 \end{cases}$$

$$h(n) = h_d(n)w(n) = \begin{cases} \frac{1}{p(n-10)} \sin[\frac{p}{2}(n-t)], & 0 \leq n \leq 20, n \neq 10 \\ \frac{1}{2}, & n = 10 \\ 0, & n \text{ 为其他} \end{cases}$$



$$h(n) = h_d(n)w(n) = \begin{cases} \frac{1}{p(n-10)} \sin\left[\frac{p}{2}(n-10)\right], & 0 \leq n \leq 20, n \neq 10 \\ \frac{1}{2}, & n = 10 \\ 0, & n \text{ 为其他} \end{cases}$$

$$h(0) = 0; h(1) = \frac{1}{9p} = 0.035;$$

$$h(2) = 0; h(3) = \frac{-1}{7p} = -0.045;$$

$$h(4) = 0; h(5) = \frac{1}{5p} = 0.064$$

$$h(6) = 0; h(7) = \frac{-1}{3p} = -0.106;$$

$$h(8) = 0; h(9) = \frac{1}{p} = 0.318;$$

$$h(10) = \frac{1}{2}$$

$$h(11) = \frac{1}{p} = 0.318; h(12) = 0;$$

$$h(13) = \frac{-1}{3p} = -0.106; h(14) = 0;$$

$$h(15) = \frac{1}{5p} = 0.064; h(16) = 0;$$

$$h(17) = \frac{-1}{7p} = -0.045; h(18) = 0;$$

$$h(19) = \frac{1}{9p} = 0.035; h(20) = 0;$$

$$(2) H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$(3) H(e^{j\omega}) = H(z) \Big|_{z=e^{j\omega}} = \sum_{n=0}^{N-1} h(n) e^{-j\omega n}$$

(4) 给出滤波器的任意一种结构实现形式
直接型



FIR滤波器设计2--往年真题

设理想数字高通滤波器的幅频响应为

$$|H_d(e^{j\omega})| = \begin{cases} 1 & p/2 \leq |\omega| \leq p \\ 0 & |\omega| < p/2 \end{cases}$$

用矩形窗函数方法设计一个 $N = 11$ 时FIR线性相位数字高通滤波器，

(1) 确定单位抽样响应序列 $h(n), n = 0, 1, \dots, N-1$

(2) 确定滤波器的系统函数 $H(z)$

(3) 确定滤波器的频率响应 $H(e^{j\omega})$

(4) 给出滤波器的任意一种结构实现形式

注：四舍五入到小数点后2位



解：理想数字高通滤波器的幅频响为

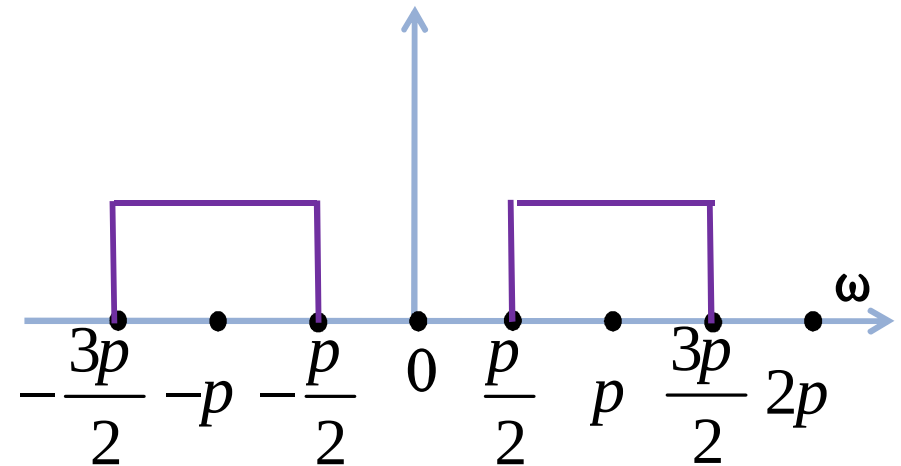
$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega a} & \omega_c \leq \omega \leq \omega_c \\ 0 & \text{其他} \end{cases}$$

$$\Rightarrow a = \frac{N-1}{2} = 5, \quad \omega_c = \frac{p}{2}$$

$$(1) |H_d(e^{j\omega})| = \begin{cases} 1 & p/2 \leq |\omega| \leq p \\ 0 & |\omega| < p/2 \end{cases}$$

$$h_d(n) = \begin{cases} \frac{1}{p(n-a)} \left\{ \sin[p(n-a)] - \sin[\omega_c(n-a)] \right\} & n \neq a \\ \frac{1}{p}(p - \omega_c) & n = a \end{cases}$$

$$= \begin{cases} \frac{1}{p(n-5)} \left\{ \sin[p(n-5)] - \sin\left[\frac{p}{2}(n-5)\right] \right\} & n \neq 5 \\ \frac{1}{p}\left(p - \frac{p}{2}\right) & n = 5 \end{cases}$$



$$h_d(n) = \begin{cases} \frac{1}{p(n-5)} \left\{ \sin[p(n-5)] - \sin\left[\frac{p}{2}(n-5)\right] \right\} & n \neq 5 \\ \frac{1}{p} \left(p - \frac{p}{2}\right) & n = 5 \end{cases}$$

$$h(n) = h_d(n)w(n) = \begin{cases} \frac{1}{p(n-5)} \left\{ \sin[p(n-5)] - \sin\left[\frac{p}{2}(n-5)\right] \right\}, & 0 \leq n \leq 10, n \neq 5 \\ \frac{1}{p} \left(p - \frac{p}{2}\right), & n = 5 \\ 0, & n \text{ 为其他} \end{cases}$$

$$h(0) = -\frac{1}{5p} = -0.064; \quad h(1) = 0; \quad h(2) = \frac{1}{3p} = 0.106; \quad h(3) = 0; \quad h(4) = -\frac{1}{p} = -0.318;$$

$$h(5) = \left((-1)^{n-5} \frac{\sin\left[\frac{p}{2}(n-5)\right]}{p(n-5)} \right) \bigg|_{n=5} = \frac{1}{2};$$

$$h(6) = -\frac{1}{p} = -0.318; \quad h(7) = 0; \quad h(8) = \frac{1}{3p} = 0.106; \quad h(9) = 0; \quad h(10) = -\frac{1}{5p} = -0.064$$

$$(2) H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$(3) H(e^{j\omega}) = H(z) \Big|_{z=e^{j\omega}} = \sum_{n=0}^{N-1} h(n) e^{-j\omega n}$$

(4) 给出滤波器的任意一种结构实现形式
直接型

FIR滤波器设计3-1--往年真题

设理想数字带通滤波器的幅频响应为

$$|H_d(e^{j\omega})| = \begin{cases} 1 & p/4 \leq |\omega| \leq p/2 \\ 0 & p/2 \leq |\omega| \leq p, |\omega| \leq p/4 \end{cases}$$

用矩形窗函数方法设计一个 $N = 9$ 时FIR线性相位数字带通滤波器，

(1) 确定滤波器单位抽样响应序列 $h(n), n = 0, 1, \dots, N-1$

(2) 确定滤波器的系统函数 $H(z)$

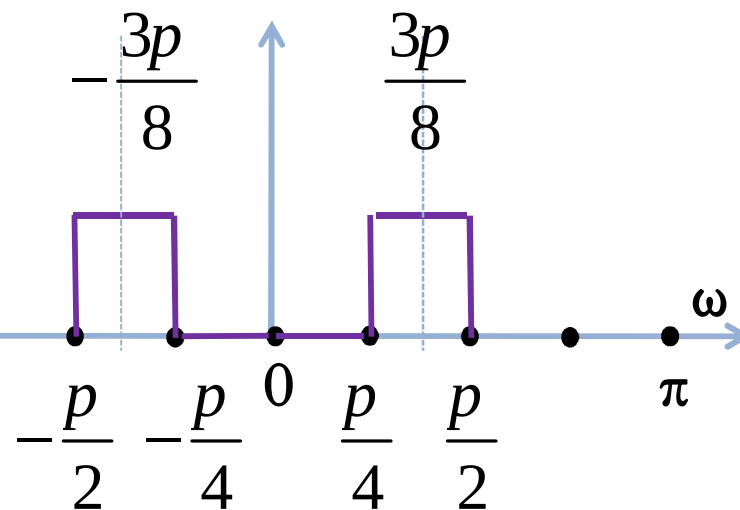
(3) 给出滤波器的任意一种结构实现形式

注：四舍五入到小数点后2位

解：理想数字带通滤波器的幅频响为

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega a} & -\frac{p}{8} \leq \omega \pm \frac{3p}{8} \leq \frac{p}{8} \\ 0 & \text{其他} \end{cases}$$

$$\Rightarrow a = \frac{N-1}{2} = 4, \quad \omega_1 = \frac{p}{4}, \quad \omega_2 = \frac{p}{2}$$



$$(1) h_d(n) = \begin{cases} \frac{1}{p(n-a)} \left\{ \sin[w_2(n-a)] - \sin[w_1(n-a)] \right\} & n \neq a \\ \frac{1}{p} (\omega_2 - \omega_1) & n = a \end{cases}$$

$$= \begin{cases} \frac{1}{p(n-4)} \left\{ \sin\left[\frac{p}{2}(n-4)\right] - \sin\left[\frac{p}{4}(n-4)\right] \right\} & n \neq 4 \\ \frac{1}{4} & n = 4 \end{cases}$$

$$h(n) = h_d(n)w(n) = \begin{cases} \frac{1}{p(n-4)} \left\{ \sin \left[\frac{p}{2}(n-4) \right] - \sin \left[\frac{p}{4}(n-4) \right] \right\}, & 0 \leq n \leq 8, n \neq 4 \\ \frac{1}{4}, & n = 4 \\ 0, & n \text{ 为其他} \end{cases}$$

$$h(0) = 0; h(1) = \frac{1}{-3p} \left(1 - \frac{\sqrt{2}}{2} \right) = -0.03;$$

$$h(2) = \frac{1}{-2p} (-1) = 0.16; h(3) = \frac{1}{-p} \left(-1 - \frac{\sqrt{2}}{2} \right) = 0.54;$$

$$h(4) = \frac{1}{4};$$

$$h(5) = \frac{1}{p} \left(1 + \frac{\sqrt{2}}{2} \right) = 0.54; h(6) = \frac{1}{2p} (1) = 0.16;$$

$$h(7) = \frac{1}{3p} \left(-1 + \frac{\sqrt{2}}{2} \right) = -0.03; h(8) = 0;$$

$$(2) H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$(3) H(e^{j\omega}) = H(z) \Big|_{z=e^{j\omega}} = \sum_{n=0}^{N-1} h(n) e^{-j\omega n}$$

(4) 给出滤波器的任意一种结构实现形式
直接型